Viewing the world systemically.

ATIS Theory Development:
Logics, Models & Theories …
Types of Systems … Research Methodologies

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Preface

There is a long history in the social sciences of researchers attempting to develop a consistent and comprehensive theory of education, theory of learning, theory of behavior, and other theories of concern to social scientists but to no avail. And, even with this failure, they persist with the following lament:

> In spite of their use of the scientific approach and accumulation of a large quantity of reliable knowledge, education and the other social sciences have not attained the scientific status typical of the natural sciences. The social sciences have not been able to establish generalizations equivalent to the theories of the natural sciences in scope of explanatory power or in capability to yield precise predictions.[1]

What they fail to recognize is the reason for this lack of theory development. It would seem that with over 100 years of failure, one would begin to look for the reasons that the achievements of the physical sciences have not been attained in the social sciences.

What we will herein determine is that first, the "scientific approach" is not even used by physical scientists, regardless of their claims to the contrary, and second, an "accumulation of a large quantity of reliable knowledge" does not result in theory.

What we will do is present an empirical theory for intentional systems that is applicable to all of the social sciences, and that the theory must be developed as an axiomatic theory.

In order to get the most out of this report, certain background is necessary. This report is presented both in a descriptive manner as well as mathematical. But for either approach, there is one work that should be read before proceeding too far with this report. That work is the seminal work prepared and published by Elizabeth Steiner on theory construction: *Methodology of Theory Building*.[2] The research provided therein is critical reading for any serious reader of this study.

Further, a background or at least a basic knowledge of logic and mathematical logic, in particular, is necessary to understand some of the more technical work in this report.

That said, this report should be of value to anyone interested in the social sciences and, in particular, one who is attempting to develop a theory within the social sciences.

A note needs to be made concerning the definition of 'theory' in the social sciences.

The definition of 'theory' in the social sciences has actually been well-defined in much the same way that is intended in this report. For example, Shutt, Professor at the University of Wisconsin, states that 'theory' is defined as follows:

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[1] Ary, D., Jacobs, L.C., and Razavieh, A., (1985), *Introduction to Research in Education*, Holt, Rinehart and Winston, New York, p. 19.

[2] Steiner, Elizabeth, (1988), *Methodology of Theory Building*, Indiana University, Educology Research Associates, Sydney.

I. What is a theory?

    A. [A theory is a] logically interrelated set of propositions about empirical reality. These propositions are comprised of:

        1. Definitions: Sentences introducing terms that refer to the basic concepts of the theory

        2. Functional relationships: Sentences that relate the basic concepts to each other. Within these we have

            a. Assumptions or axioms

            b. Deductions or hypotheses

        3. Operational definitions: Sentences that relate some theoretical statement to a set of possible observations

    B. Why should we care? What do theories do?

        1. Help us classify things: entities, processes, and causal relationships

        2. Help us understand how and why already observed regularities occur

        3. Help us predict as yet unobserved relationships

        4. Guide research in useful directions

        5. Serve as a basis for action. "There is nothing so practical as a good theory."

    http://www.ssc.wisc.edu/~jpiliavi/357/theory.white.pdf

Also, we have: 'theory' in the social sciences has been defined as:

    A theory is a set of interrelated concepts, definitions, and propositions that explain or predict events or situations by specifying relations among variables. (Office of Behavioral and Social Sciences Research, New England Research Institutes, esource@neriscience.com.)

Whereas in this report there may be some variations in terminology, the above definitions essentially state what 'theory' is whether in the social sciences or the physical sciences. And, as essentially stated above concerning what theories do for us, in this report it will be seen what the purpose of a theory is:

The purpose of a theory is to provide the means to develop mathematical, analytical, or descriptive models that predict counterintuitive, non-obvious, unseen, or difficult-to-obtain outcomes.

Whereas in the social sciences, the "mathematical" development of a theory is normally restricted to statistical measures, in this report we focus on a theory being axiomatic. The notion of theory has not changed, just the type of mathematics used to define the theory. In all fields it is contended that a theory should be "analytical", although the type of analysis may be different. In the social sciences descriptive models are the norm when applying a theory to empirical observations. In this report, we argue for more logico-mathematical applications.

After developing the type of theory to be utilized, it is then seen that the purpose is to be able to "*predict counterintuitive, non-obvious, unseen, or difficult-to-obtain outcomes.*" The point here is simply that if an outcome is already known or easily discernable, then there is no need for the theory. A theory may be able to be developed, but for what purpose if the outcome is already known? If you already know that a student will not learn mathematics if the student is not taught mathematics, then what is the purpose of designing a theory that predicts just such an outcome?

What is "*ATIS*"?

ATIS, *Axiomatic Theory of Intentional Systems*, is an *axiomatic formal empirical theory* designed specifically for *intentional systems*. Therefore, to understand what *ATIS* is, we must understand the following:

(1) What is an "*Axiomatic Formal Empirical Theory*"?

(2) What is a "*Theory Model*"?

(3) What is an "*Intentional System*"?

As can be seen, there are numerous questions that must be answered if we are to clearly understand what it is that *ATIS* is and how it is applied to help us answer questions relevant to our lives.

In order to properly develop our understanding, this study will be presented in both a descriptive manner and a very technical manner that relies on various mathematical logics and formalisms not generally found in studies of this type. The reason for this reliance on a logico-mathematical development is so that there will be no confusion as to what is intended and so that the argument for *ATIS* can be clearly followed. While descriptive theories are of value for initial understanding, they also allow for interpretations that are not always consistent, and hence the numerous debates for understanding found in the social sciences.

A more rigorous and formal approach helps to eliminate confusion. And, a logico-mathematical axiomatic approach reduces that confusion even further. In this study, we will use a combination of both the descriptive and the formal so that we will have not only an intuitive understanding of what is intended by way of the descriptive, but also a very precise understanding by way of the formal.

So, let us revisit our questions stated above.

What is an "*Axiomatic Formal Empirical Theory*"?

'**Axiomatic**' is simply a theory that is founded on axioms rather than description.

'**Formal**' means that logico-mathematical symbolisms are used.

'**Empirical**' means that our concern is with what is verifiable by observation or experience.

'**Theory**' will be discussed in detail below

'***Axiomatic formal empirical theory***' is an empirical theory that is developed using axioms and logico-mathematical terminology.

What is a "*Theory Model*"?

The answer to this question will require an in-depth study of both '***theory***' and '***model***'.

There is much more that must be discussed before we can have a clear understanding of just how '***theory***' and '***model***' are being used in that there are many interpretations of each. To address these issues, we will refer to a seminal work prepared and published by Elizabeth Steiner on theory construction: *Methodology of Theory Building*.[3]   The research provided therein is critical reading for any serious reader of this study. What is presented here will be but a summary of the critical points she presents that are directly relevant to the questions of concern here.

What is '*Theory*'?   Steiner states the following:

> 'Theory' is derived from the Greek '*theoria*' which means contemplation or speculation. In a popular sense then one's theory is one's speculation or conjecture about something. …   However, such a popular sense does not catch up the technical sense of 'theory' in 'Einstein's theory of relativity', …[4]

In this study, we are concerned only with the technical, or formal, sense of 'theory'.

Due to the importance of understanding the meaning of a technical or formal theory in the social sciences, a fairly extensive discussion is provided below. This discussion is also provided due to the mischaracterizations of 'theory' that can be found in the literature of the social sciences, and the wide range of interpretations of 'theory'.

Due to their great number, we will begin by taking a look at *Theories of Learning*.

Theories of Learning

Overview

Before considering the nature of theory development, we will first take a look at the *theories of learning* found in the literature. There are a multitude of *theories of learning* in the social sciences. And, it will be seen from these theories why there has never been, and <u>cannot be</u>, any general theory of learning obtained as a result of these methodologies of theory development.

All of these attempts at theory development are describing "what is" rather than asserting the general principles that "*result* in what is"—hence, the use of hypotheses. Hypotheses help to answer questions about "what is, "<u>but do not provide a means to determine "general principles</u>".

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[3] Steiner, Elizabeth, (1988), *Methodology of Theory Building*, Indiana University, Educology Research Associates, Sydney.
[4] Ibid, p. 4.

As will be seen, hypotheses are a means to inductively validate assertions, but validation does not provide the means to develop the basic assumptions or general principles upon which comprehensive, complete and consistent theories are founded.

Simply put, hypotheses are not designed to develop theory; they are designed to validate theory. But, when the entire research methodology of the social scientist is hypothesis-driven, then there is no means by which legitimate theory can be devised since, by use of the hypothesis-methodology, there is no intent to devise such a theory.

Current Theories of Learning—A Review of the Literature

As noted above, there are a multitude of *theories of learning*. Some of these are listed below. And with each it will be seen that what is attempted is not to establish general principles, but to describe the learning process as perceived from a narrow perspective. For example, consider the following list of theories:

Adult Learning Theory	Mathematical Learning Theory
Anchored Instruction Theory	Operant Conditioning Theory
Contiguity Theory	Repair Theory
Constructivist Theory	Script Theory
Conditions of Learning Theory	Sign Learning Theory
Dual Coding Theory	Situated Learning Theory
Experiential Learning Theory	Structural Learning Theory
Functional Context Theory	Subsumption Theory
Genetic Epistemology Theory	Symbol Systems Theory
Information Pickup Theory	Triarchic Theory
Information Processing Theory	

Each theory is framed as a hypothesis that can be tested, rather than a presentation of general principles that include the observations and produce unknown outcomes. Each theory treats learning from a different perspective and introduces hypotheses to validate that perspective.   For example, consider the following learning theories:

Adult Learning Theory, *as the name suggests, is restricted to considering the characteristics of adult learners. Then, the analysis of these learning characteristics is divided into* personal characteristics *and* situational characteristics. *However, as will be discussed in this report, such analysis does not lead to theory. What such analyses do is develop the means to interpret what one observes, possibly by the abduction of mathematical models from mathematics, or behavioral models from psychology.* Adult Learning Theory *is actually* Adult Learning Laws *that can be used to assist one in developing adult learning programs by describing learning events or empirical observations.* Adult Learning Theory *does not design theory; it simply establishes predetermined guidelines that are considered of value for the instruction of adults.*

Anchored Instruction Theory *is an* ***instructional methodology*** *that has demonstrated success in helping students to learn by utilizing "interactive videodisc technology." While it is a paradigm for others to emulate in designing problem-focused learning, it is not a theory in that no logically-derived hypotheses are obtained or possible. Anchored Instruction Theory does not devise theory; it simply provides a predetermined framework that is considered of value for instruction that can be implemented in various learning programs by describing learning events or empirical observations.*

Contiguity Theory *is framed as a hypothesis: "A combination of stimuli which has accompanied a movement will on its recurrence tend to be followed by that movement." This is an assertion that can be validated through testing. But such validation does not devise theory; it simply establishes that under the given conditions a specific "movement" will be obtained by describing learning events or empirical observations.*

Constructivist Theory *provides guidelines by which it is believed students will learn; for example, "The instructor should try ...," "The instructor and student should engage ...," etc. This is but a prescription for learning, and not a theory for discovery of learning processes in that the conclusion is already known. The four major aspects of learning provided are prescriptive of what should be done for a student to learn. These prescriptions do not devise theory; they simply guide one to assist in establishing a predetermined learning environment that it is assumed will be effective by describing learning events or empirical observations.*

In each of these cases, theory is not being devised; only guidelines that can help one to design a more effective learning environment—that is, *they describe known learning events or empirical observations*. This is the problem with hypothesis-driven methodologies—whereas one may validate that a specific learning methodology may be effective, there is nothing that has been validated that can help to devise theory that can then be used to predict outcomes even in similar environments. Every learning environment must be re-validated to determine if the hypothesis is valid.

But, what no one has done is what is critical to the development of a general theory of learning—devise a theory that encompasses all of these perspectives; devise the *basic elements* that will bring under it all of the hypotheses that have been proposed and validated. And, the reason that this has not been done lies right at the foot of the methodology being used—hypothesis-driven methodologies. Each researcher starts out anew with a new perspective and validates it as a result of a new hypothesis. This methodology must change if there is to ever be a theory of learning founded on general principles that is comprehensive and consistent.

Now, there is no question that the research provided by these hypotheses has resulted in understanding of the learning process and of student development or learning. It is certainly recognized that a vast wealth of information and understanding has been obtained. The value and results of all of the research is not in question. The concern is that all of this research has been designed for just one thing—to amass all of this wealth of information and understanding for its own sake, but not to develop any general principles about learning processes and student development that could be confidently applied to diverse classes of students. With great understanding, exceptional learning programs can be developed within a specific class of students, but no predictive outcomes can be obtained as to just how valuable the learning program is since there is no theory founded on general principles. The great diversity of the learning theories that are available can be seen by those cited above.

And, there is a further problem that results from identifying the above hypotheses as *theories*—when *theory* is reduced to include every *conjecture*, then *theory* is a term that is of limited to no value. When *theory* can include the assertions of one whose only purpose is a political or financial agenda, as well as those that are carefully and logically-developed descriptive or axiomatic theories, then we as scientists have no one to blame but ourselves when the religiously-motivated or financially-motivated, or otherwise personally-motivated layman asserts that something is "only a theory," implying that it is of little or no value and the pronouncements of a layman concerning the scientific endeavor are just as important and credible as the professional scientist.

Unfortunately this perspective is furthered when texts on *Scientific Research in Education* mislead one on the nature of just such research. The following text edited by Richard J. Shavelson and Lisa Towne purports to present an overview of educational research and theory development, but falls substantially short by not helping to clarify the problems within education research. Of course, as editors, they were but compiling the results of a committee that had been:

> Assembled in the fall of 2000 and was asked to complete its report by the fall of 2001. The charge from the committee's sponsor, the National Educational Policy and Priorities Board of the U.S. Department of Education, was as follows:
>> This study will review and synthesize recent literature on the science and practice of scientific education research and consider how to support high quality science in a federal education research agency.[5]

The problem, of course, does not lie with the editors, but with the entire education research community. The problem lies with the very assumptions about the development of education as a science and the manner in which "knowledge in education accumulates." The focus of research in education, as perceived by the committee, was stated as follows:

> • How can research-based knowledge in education accumulate?

> The committee believes that rigor in individual scientific investigations and a strong federal infrastructure for supporting such work are required for research in education to generate and nurture a robust knowledge base. Thus, in addressing this question, we focused on mechanisms that support the accumulation of knowledge from science-based education research—the organization and synthesis of knowledge generated from multiple investigations. The committee considered the roles of the professional research community, the practitioner communities, and the federal government. Since *we view the accumulation of scientific knowledge as the ultimate goal of research, this issue weaves throughout the report*.[6] [Emphasis added.]

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[5] Shavelson, Richard J. and Towne, Lisa, Eds. (2002). *Scientific Research in Education*, National Educational Policy and Priorities Board of the U.S. Department of Education, p. 22.

[6] Ibid. p. 24.

The "goal of research" as the "accumulation of scientific knowledge" clearly states the problem in education research today. As will be discussed below, even Ary, et al., as noted previously, recognize that even with the "*accumulation of a large quantity of reliable knowledge, education and the other social sciences have not attained the scientific status typical of the natural sciences.*"[7] They, and most others, totally miss the point that scientific knowledge as found in the natural sciences is not and cannot be obtained by simply accumulating "a large quantity of reliable knowledge." Unfortunately, "science" as "accumulation of knowledge" determines the basis for the committee's report as reflected by the title of the second chapter—"Accumulation of Scientific Knowledge"—which confirms that they totally misunderstand the nature of the scientific endeavor. The first paragraph of the report states the problem, but also the basis for this Committee's agenda—to refute the obvious. They assert:

> The charge to the committee reflects the widespread perception that research in education has not produced the kind of cumulative knowledge garnered from other scientific endeavors. Perhaps even more unflattering, a related indictment leveled at the education research enterprise is that it does not generate knowledge that can inform education practice and policy. The prevailing view is that findings from education research studies are of low quality and are endlessly contested—the result of which is that no consensus emerges about anything.[8]

Unfortunately, rather than take this perception at face value, the Committee tries to refute it. An in-depth analysis of this Committee's report might be appropriate except that the Committee itself provides the answer that the report is prepared to support an agenda for <u>Federal funding(!)</u> rather than to present an unbiased report about the nature of research and the reason for the lack of any real theory development in education that would provide a basis for "knowledge accumulation" in education. In response to the above perception of education research, the Committee states:

> Is this assessment accurate? Is there any evidence that scientific research in education accumulates to provide objective, reliable results? Does knowledge from scientific education research progress as it does in the physical, life, or social sciences? To shed light on these questions, we consider how knowledge accumulates in science and provide examples of the state of scientific knowledge in several fields. In doing so, we make two central arguments in this chapter.

> First, research findings in education have progressed over time and provided important insights in policy and practice. We trace the history of three productive lines of inquiry related to education as "existence proofs" to support this assertion and *to convey the promise for future investments in scientific education research*. What is needed is more and better scientific research of this kind on education.

> Our second and related argument is that in research across the scientific disciplines and in education, the path to scientific understanding shares several common characteristics. ... The path to scientific knowledge wanders through contested terrain as researchers, as well as the policy, practice, and citizen communities critically examine, interpret, and debate new findings and it *requires substantial investments of time and money*.[9] [Emphasis added.]

Unfortunately, again, the Committee now elicits the input of the "citizen community" to "critically examine new findings" of the education research community. We continue to contend with the results of just such a position as the "citizen community" bringing to education their own personal agendas disguised as "scientific inquiry"; e.g., the Creationists. When every scientific inquiry is argued in the court of public opinion **and bias**, then education researchers, or at least this Committee, has brought on itself the problems of scientific inquiry experienced centuries ago. It is true that the religiously-motivated public has replaced the religious clerics who previously mandated "scientific" outcomes to assure their personal agendas as with the findings of Galileo. What was Galileo's response?

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[7] Ary, Donald; Jacobs, Lucy Cheser; and Razavieh, Asghar. (1985). *Introduction to Research in Education*, p. 19.
[8] Op cit., Shavelson, p. 28
[9] Ibid. p. 29.

Due to the clear political and financial agenda being pursued by this Committee, little more needs to be said. However, a few points will be made as such will help to clarify just exactly what does have to be done in education research.

To support its position concerning research in education, the Committee tries to rely on the development of research in molecular biology. They assert:

> The earliest model of the gene was derived from Mendel's pea plant experiments in the 1860s.[10]

And the science related to molecular biology and the modern concept of the gene developed from there. The problem with this purported analogy is that the science of molecular biology did in fact build on previous discoveries, each building on the developing theory—unlike any corresponding development in education. **There is no basic theory of education on which to build**.

In Chapter 3, the Committee states:

> In Chapter 2 we present evidence that scientific research in education accumulates just as it does in the physical, life and social sciences. Consequently, we believe that such research would be worthwhile to pursue to build further knowledge about education.[11]

Unfortunately, such is just not the case. First, the Committee seems to believe that education is not part of the social sciences. The social sciences, of course, have the same problems in research as are found in education—there is no basic theory on which to build, and both are furthered by hypothesis-driven research methodologies that cannot devise theory.

More important, the Committee seems to have no idea how scientific research "accumulates" in the physical sciences. This will be discussed in more detail below; however, for now it must simply be recognized that the physical sciences, and physics, in particular, are furthered as the result of well-defined theories. Education and the social sciences have none.

While Shavelson and Towne have provided some insight concerning the problems in education research today, John W. Creswell (Creswell, 2003) provides additional insight concerning those problems. Most of the problems center on a misunderstanding of the nature of theory and the resulting name-calling, especially with the degradation of the *Post-Positivists*.

The easiest to discern, possibly, is the assertion that those taking an "advocacy/participatory approach" are conducting "research" to develop "knowledge."[12] Creswell states:

> This position arose during the 1980s and 1990s from individuals who felt that the postpositivist assumptions imposed structural laws and theories that did not fit marginalized individuals or groups or did not adequately address issues of social justice. ... In the main, these inquirers felt that the constructivist stance did not go far enough in advocating for an action agenda to help marginalized peoples. These researchers believe that inquiry needs to be intertwined with politics and a political agenda. Thus, the research should contain an action agenda for reform that may change the lives of the participants, the institutions in which individuals work or live, and the researcher's life. ... Therefore, theoretical perspectives may be integrated with the philosophical assumptions that construct a picture of the issues being examined, the people to be studied, and the changes that are needed. Some of these theoretical perspectives are listed below.

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[10] Ibid. p. 31.
[11] Ibid. p. 50.
[12] Creswell, 2003, p. 9.

    • Feminist perspectives.   • Queer theory.
    • Racialized discourses.   • Critial theory.
    • Disability inquiry.[13]

As has been adequately demonstrated by Marx and his ilk, the October Revolution, etc., advocacies of such political agendas are not doing science but propaganda and fear-mongering in the guise of "social justice." By definition of the special interests, this does not define either "research" or "pursuit of knowledge," but simply the attempt to impose one's personal agenda on others who may or may not wish to support the agenda. Even if there is support for the agenda, such still does not bring it under any "pursuit of knowledge" nor "research," but it remains a personal agenda for the purpose of imposing one's will on another.

To claim, as in the last sentence of the above quote, that these perspectives are "theoretical perspectives" is to intentionally mislead those laymen who do not know better. These perspectives are nothing more or less than personal agendas. Today, one such perspective is "creationism" strutting itself as "science." To include these agendas under the guise of science is to do a great disservice to all scientists and serious researchers in education.

As the advocacy/participatory approach apparently grew out of the constructivist stance as one that did not go far enough, we can now go back and have a better understanding of the constructivists. According to Creswell:

> Assumptions identified in these works [by Lincoln and Guba, Schwandt, Neuman, and Crotty] hold that individuals seek understanding of the world in which they live and work.[14]

Well, of course they do, but that does not mean that they are creating theory. It is uncertain whether or not this assertion by Creswell is to be taken as something profound. He continues:

> They develop subjective meanings of their experiences—meanings directed toward certain objects or things. These meanings are varied and multiple, leading the **researcher** to look for the complexity of views rather than narrowing meanings into a few categories or ideas. The goal of research, then, is to rely as much as possible on the participants' views of the situation being studied.[15] [Emphasis added.]

And, we wonder why there is no theory of education! When everyone is a "researcher", then no one is. And to assert that the "*goal of research*" is to "*rely as much as possible on the participants' views*" indicates a total lack of understanding of what a researcher does or the process of scientific discovery.

Of course, normally science is intended to devise a means to mitigate the influence of subjective evaluation so as not to prejudice the outcomes. But, by relying "*as much as possible on the participants' views*" the outcome of such research is clearly designed to obtain the **biased** results of the "researcher" and to further a personal agenda. The advocacy/participatory advocates simply make this intent more clear.

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[13] Ibid., pp. 9-10.
[14] Ibid., p. 8.
[15] Ibid., p. 8.

Most telling concerning the constructivist approach is the following:

> Rather than starting with a theory (as in post-positivism), inquirers generate or inductively develop a theory or pattern of meaning.

For example, in discussing constructivism, Crotty (1998) identified several assumptions:

1. Meanings are constructed by human beings as they engage with the world they are interpreting. Qualitative researchers tend to use open-ended questions so that participants can express their views.

2. Humans engage with their world and make sense of it based on their historical and social perspective .... They also make an interpretation of what they find, an interpretation shaped by the researchers' own experiences and backgrounds.[16]

The first question that must be addressed concerning the first point above is: To what end? Are "researchers" simply providing a venue in which anyone can share their own ignorance?

And, the response to the second point is: That is exactly why, in legitimate research, one attempts to preclude personal bias. Of course, the Flat-Earth Advocates do need an outlet, and the constructivists have provided one. But that should not be confused with scientific inquiry, except for the sociologist who wants to determine just what it is that makes these people tick.

As shown in this study, much of the attraction to such advocacies as discussed above rests directly with the problem of comprehending just what a theory is and what is the nature of scientific inquiry. But, it also may simply be a result of the personal agendas of the advocates as discussed above. In those cases, reason is not the issue and there can be no legitimate dialog.

Hypothesis-Based Research Methodologies

Now let us address the current research methodologies in the social sciences. As seen from the list of learning theories presented above, the social sciences, in particular, have relied on hypothesis-driven research to arrive at decisions concerning their industry. As will be seen, however, such research methodology cannot result in predictive outcomes beyond the specific case evaluated. Whereas validated hypotheses may help make decisions concerning narrowly-defined problems, they do not provide a basis for predicting outcomes under differing conditions.

Before considering various hypothesis-based methodologies, we will consider the methodology required for theory construction that has been clearly explicated by Charles Sanders Peirce in 1896 and Elizabeth Steiner in 1988. From Peirce (Peirce, 1896), we have:

> **§10. KINDS OF REASONING**
>
> 65. There are in science three fundamentally different kinds of reasoning, Deduction (called by Aristotle συναγωγή or αναγωγή), Induction (Aristotle's and Plato's ἐπαγωγή) and Retroduction (Aristotle's ἀπαγωγή), but misunderstood because of corrupt text, and as misunderstood usually translated *abduction*. Besides these three, Analogy (Aristotle's παραδειγμα) combines the characters of Induction and Retroduction.[17]

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[16] Ibid., p. 9.

[17] Peirce, Charles Sanders, (1896), notes from "History of Science" (not published), In: *Collected Papers of Charles Sanders Peirce, Principles of Philosophy*, (ed.) Hartshorne, C. and Weiss, P., The Belknap Press of Harvard University Press, Cambridge, Massachusetts (1960).

And Steiner puts the issue into clear perspective as follows (Steiner, 1988):

- Retroduction devises theory.
- Deduction explicates theory.
- Induction evaluates theory.

To these three modes of theory development, there is a fourth as alluded to by Peirce—abduction.

- Abduction extends theory.

To see just what each of these modes of theory development do, consider the following:

- ***Retroduction*** is the logical process by which a point of view is utilized to devise a conjecture or theory; whether that view is determined by utilizing an existing theory to devise a new one, or is obtained from the *whole cloth of relevant knowledge* by which perspectives from several relevant fields of knowledge are utilized to devise a new theory.
- ***Deduction*** is the logical process by which a conclusion (theorem) is obtained as the implication of assumptions and previous theorems. These derivations are strictly logical. It is these derivations that argue for an axiomatic theory, since it is only axiomatic theories that have the structure required whereby an outcome can be used to encourage acceptance of the results of the research independent of subjective argument.

- ***Abduction*** is the logical process by which a theoretical construct of one theory is utilized to analyze or interpret the parameters of another theory. This is frequently used by using mathematics to develop *mathematical models* to interpret observations of a given theory.

- ***Induction*** is the logical process by which theory is evaluated. This is the testing of a theorem (*logically-derived hypothesis*) by which empirical observations are obtained to confirm or reject the theorem/hypothesis.

The problem with hypothesis-driven methodologies has been recognized in the social sciences by various researchers, but no alternative has been generally accepted. Essentially, social scientists have defined-themselves-out of theory development as an alternative, since they keep attempting to refine a methodology that cannot devise theory.

Theory Construction in the Social Sciences

To understand the place of theory in the social sciences, it is instructive to review the different interpretations of theory that have been proposed. In particular, it is critical to understand that hypotheses do not result in theory and the almost total reliance of the social sciences on hypothesis testing is the primary reason why there are no generally accepted theories in the social sciences, and none that are comprehensive, consistent, complete and axiomatic.

Probably the biggest problem that social scientists have to confront is the prevailing methodology of classical science. Classical science is dependent on the following techniques for the development of theory: observation, hypothesis, and experiment.   This is an inductive process, and one that is counter to what Peirce and Steiner have clearly analyzed. Both before and after Peirce, and before and after Steiner, the classical development of theory in the social sciences has been attempted; that is, the process of induction.

The reason that the social scientist has relied on induction and the resulting hypothesis-driven research methodology is that the social scientist interpreted the physical scientist as having developed theory by just such a method. However, such has been misguided. It is instructive to recognize that a physicist may consider the development of theory to be that of the classical science, even though it is not. Since a physicist is not so much concerned with the process of theory development as with the development of theory, this confusion is understandable. Therefore, while a physicist may assert that the development of theory is by induction, as claimed for Rock Theory (discussed below), in fact physicists developing such theory proceed in a manner defined by Peirce and Steiner as retroduction in the vertical development of new theory. Therefore, when a social scientist attempts to apply the theory-development methodology of physics; for example, as defined by a physicist, the social scientist may believe what the physicist asserts— that the methodology is one of induction. Hence, the inabilities of the social scientist to in fact develop theory—the wrong methodology is being applied.

This misinterpretation of theory development in the physical sciences has led to some of the following attempts to design a process by which theory can be developed in the social sciences.

<u>Karl Raimund Popper</u>. It is instructive to note that Karl Popper recognized the problems with the classical approach to the development of theory. As an alternative, Popper proposed a new scientific methodology. In his two books, The Logic of Scientific Discovery[18] *and* Conjectures and Refutations[19]*, he introduced an alternative to inductive inference for theory building—*hypothetico-deductive scientific method for theory development*.

While this approach may appear to be better than the inductive method, it falls short of clearly defining a methodology that will result in scientific theory. In fact, it but jumps to the hypothesis and explicates the "theory" from there. 'Hypothetico-deductive' is simply a process whereby we deductively determine outcomes from a hypothesis, but the hypothesis is not theory. And an outcome derived from a hypothesis that is not founded in theory is without foundation. As a result the tested hypotheses are just statements created by a researcher for the sole purpose of carrying out an experiment comparable to the classical approach they were to replace. In fact, this is so even if the hypothesis is a deduction from another hypothesis. Deductive inferences are no more reliable than the hypotheses upon which they are founded when the hypothesis is not derived from axioms, basic assumptions.

The difference between the physical and other mathematical sciences and the social sciences is that in physics and the other mathematical sciences, there is an underlying theory upon which all hypotheses rely. In the social sciences there is none. It is not that there is a distinction between what the scientists in the physical sciences and social sciences define as 'theory', it is just that in the physical sciences a theory of physics, etc., has actually been developed. So the question is simply, why has no such theory been developed in the social sciences, and in education, in particular?

In physics the researcher proceeds from an existing theory, whether that is Newtonian Physics, Einstein Physics, the Kinetic Theory of Gases, Thermodynamics, or some other theory, and this theory provides the framework in which the scientist works. Hypotheses in physics are in fact derived from an existing theory.

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[18] Popper, K., (1961), *The Logic of Scientific Discovery*, Basic Books, New York.

[19] Popper, K., (1963), *Conjectures and Refutations*, Routledge & Kegan Paul, New York.

Then, ***the <u>predictions</u> derived therefrom determine the value of the theory***. This is the critical point relating to theory construction—its main purpose is to *predict*. Further, the predictions of the theory provide new outcomes that no intuition or hypothesis could have predicted. While hypotheses in the social sciences are designed to state what someone believes to be true, and, clearly, cannot state anything that the designer cannot conceive, the purpose of a theory is distinctly different as follows, and is so stated by Shutt above. Further, the main point here is that while some statements of a theory may in fact be a "theory", if they do no more than state the obvious, the "theory" is of little value. With this in mind, we have the following:

> **Purpose of a Theory**—If there are no counterintuitive results derived from a theory, or if there are no predictions from the theory that are not obvious, or if the theory does not provide outcomes that were not seen, or if the theory does not obtain results that are otherwise difficult to obtain, then there is no need for the theory.
>
> Predictions from a theory are a result of equations (mathematical models) or logical derivations developed from the theory and such equations or logical schema does not rely on any preconceived notions that the effect could even exist.
>
> Therefore, ***the purpose of a theory is to provide the means to develop mathematical, analytical, or descriptive models that predict counterintuitive, non-obvious, unseen, or difficult-to-obtain outcomes***.

When all we are testing are outcomes that are preconceived, then we are missing the very purpose of scientific inquiry—to determine what it is that we do not know, rather than that which we have just not yet confirmed, or patterns that we have just not yet discerned.

Confirmation of a hypothesis may be interesting and of limited value, but to call a body of knowledge that does nothing more than confirms perceptions of known events is to trivialize the notion of theory to the point where any proclamation becomes a *theory*. That this is done all too frequently is confirmed by the "Charles's Law Theory" asserted by Travers as discussed below.

In fact, it is interesting to note that Popper (Popper, 1963) asserts, yet does not recognize the methodology of theory development when he cites this very <u>**non**</u>-hypothetico-deductive example:

> We all were thrilled with the result of Eddington's eclipse observations which in 1919 brought the first important confirmation of Einstein's theory of gravitation.[20]

Clearly, Einstein's theory was not devised as a result of any observation. The observation was initiated as a direct result of the deductive inferences of the theory. Einstein's "hypothesis" concerning light and gravitation was obtained deductively from his theory; that is, it was a *logically-derived, theory-based hypothesis*, and not from some hypothesis founded on an observation. A hypothesis is not a theory, and this example by Popper refutes the very nature of scientific discovery that Popper claims.

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[20] Ibid., p. 34.

Robert Morris William Travers.   *While others before him recognized the problems with the classical approach to the development of theory, Robert Travers embraces it completely to the point of asserting that theory is developed directly from the data of observations. In 1972, R.M.W. Travers published a book entitled* An Introduction to Educational Research[21]. *Therein, Travers states:*

> In the behavioral sciences, one common practice is for the scientist to develop theories that postulate underlying mechanisms to account for behavior as it is observed. ... These imaginary mechanisms are known as *constructs*. (pp. 14-15)
> The problem that is eventually isolated may be stated in terms of a question for which the proposed research is designed to obtain an answer. Sometimes the question to be answered is referred to as a hypothesis. (p. 81).
> **It will be assumed in this discussion that the hypothesis is firmly rooted in a framework of theory.** (p. 81) [Emphasis added.]

Travers confirms that research in the behavioral sciences as practiced is concerned with explaining observed behavior, rather than developing theories that encompass such behavior. This is an important distinction. Theories encompass observations; they guide the researcher to look in certain directions; they inform the researcher about outcomes that were never conceived; most important, they do <u>not</u> just reflect what someone observes.

First, it must be established beyond doubt but that theory is not derived from observations, and, in particular not from any collection of data. Observation may suggest phenomena for which a legitimate theory could assist in predicting outcomes, but the theory itself must come from some other source. For example, once it is proposed from observation that the earth travels in an orbit around the sun, and is then confirmed by further observation, a question might be: What keeps the earth in this orbit rather than traveling off into space?

It is clear that no "gravity waves" were observed, and, therefore, no empirical data identifying gravity is available by which some "theory" could be derived that would describe *gravity*. The theory is clearly derived from some other means—*it is the imagination and creative insight of the innovator by which theory is developed*. Social scientists seem to miss this very important aspect of theory development. Theory is developed as the result of personal insight, and not by some mechanical means by which observations are classified into patterns for analysis.

With respect to gravity, we start with *Newton's Law of Universal Gravitation*—a statement about the relationship between bodies. This statement is not a theory; it simply defines mathematically what can be observed concerning the "gravitational attraction," the effect of the construct called "gravity," between two physical bodies. It tells us nothing about what gravity is. Further, there are numerous theories of gravity. One of the more well-known theories is *Einstein's General Relativity Theory of Gravity* from which *Newton's Law* can be derived. Other theories of gravity include: The *Dynamic Theory of Gravity*, the *Inertial Theory of Gravity*, and the *String Theory of Quantum Gravity*. It should be clear that none of these, especially *String Theory*, were obtained by collecting data.

The "isolated problem" cited by Travers is posed as a hypothesis. Surprisingly, Travers asserts that "*the hypothesis is firmly rooted in a framework of theory*." But, what theory is he referring to? Travers, quoting F.N. Kerlinger, defines 'theory' as follows:

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[21] Travers, R.M.W., (1972), *An Introduction to Educational Research*, The Macmillan Company, New York.

> A theory may be defined as "a set of interrelated constructs (concepts), definitions, and propositions that presents a systematic view of phenomena by specifying relations among variables, with the purpose of explaining and predicting the phenomena"[22].
>
> Theories knit together the results of observations, enabling scientists to make general statements about variables and the relationships among variables. For example, it can be observed that if pressure is held constant, hydrogen gas expands when its temperature is increased from 20° to 40°C. It can be observed that if pressure is held constant, oxygen gas contracts when its temperature is decreased from 60° to 50°C. A familiar theory, Charles's Law, summarizes the observed effects of temperature changes on the volumes of all gases by the statement "When pressure is held constant, as the temperature of a gas is increased its volume is increased and as the temperature of a gas is decreased its volume is decreased." The theory not only summarizes previous information but predicts other phenomena by telling us what to expect of any gas under any temperature change.[23]

Travers, like many before and after him from the social sciences, attempts to rely on theory construction in the physical sciences, and physics, in particular, to justify his vision of how theory is developed. Unfortunately, such vision is tainted by a misinterpretation resulting from a misunderstanding of just how theory in physics is actually developed. While it is legitimate to use theory from physics as a paradigm for theory construction in the social sciences, when that paradigm is misunderstood, legitimate theory in the social sciences is compromised.

Travers asserts:

> Theories knit together the results of observations.

As cited previously, it is clear that theories of gravity were not obtained as the result of observing gravity and *Newton's **Law*** is not theory.

Newton's Law may have "knit together the results of observations," but theories of gravity were not derived for such knitting; they were derived for something far more substantive—to explain what gravity is in terms of predicting the effects of such gravity, such as the bending of light rays. The bending of light rays was not observed until a theory of gravity was developed that predicted such light ray bending. The observation confirmed the theory; the theory was not derived to somehow explain an observation. In terms of Steiner and Thompson:

- Retroduction devises theory—the Theory of Relativity was devised.
- Deduction explicates theory—it was explicated that light rays bend in the presence of large gravitational fields.
- Abduction extends theory—mathematical models assisted in extending and explicating the Theory of Relativity. Who does not recognize $e = mc^2$?
- Induction evaluates theory—the empirical event of a solar eclipse was used to evaluate whether or not light rays actually bend.

By citing *Charles's Law* as an example of what behavioral scientists consider as "theory," Travers confirms that the behavioral scientist is not concerned with the development of theory—*Charles's Law* is <u>not</u> a "theory." "Knitting together results of observations" does not develop theory.

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[22] Kerlinger, Fred N., (1973). *Foundations of Behavioral Research*, 2nd ed., New York: Holt, Rinehart and Winston, p. 9.
[23] Ibid., p. 15.

Charles's Law determines specific ratios of certain empirical events, a process of abduction whereby mathematics is used to help interpret observed events. It does not design a theory concerning such events; it simply establishes equations by which such events can be measured.

The *Ideal Gas Law* is a generalization of both *Boyle's Law* and *Charles's Law*. The *Kinetic Theory of Gases* encompasses the *Ideal Gas Law*.

For example, although the *Kinetic Theory of Gases* describes the motion of many particles and how the kinetic energy of those particles produces an averaged effect of pressure, its axioms were not obtained or predicted by observing the rising of a balloon filled with gas, as was *Charles's Law*. The three assumptions upon which the *Kinetic Theory of Gases* is based are:

- Matter is composed of small particles (molecules or atoms).
- The particles are in constant motion.
- When the particles collide with each other, or with the walls of a container, there is no loss of energy.

These axioms are assumed from general considerations of matter, and not the specific filling of a balloon with gas. Further, these axioms were not obtained as the result of any observations; they were presumed as the result of the *creative insight of the researcher*. At the time that these axioms were propounded, neither molecules nor atoms could be seen. It was simply *assumed* that they existed. And, there certainly is no way to determine whether or not these particles are ever at rest, although the *preponderance of evidence* indicates that they are not. And, likewise there is no way to determine if in fact there is never any loss of energy, it is simple assumed that there is none. However, even today, atoms in gases cannot be seen, only detected and only atoms in solids can typically be seen.

It is patent that these axioms were not obtained as the result of "observation" of any empirical event.

The *Theory of Thermodynamics* is the theory of physics that encompasses the *Kinetic Theory of Gases*. Therefore, *Charles's Law* is not a theory, but an explication of the *Theory of Thermodynamics* or its sub-theory, the *Kinetic Theory of Gases*.

These theories, while explaining certain empirical observations such as those relating to gases, were not developed as a result of Charles's observations; they were developed to explain the behavior of large volumes of particles in gases.

Once again, the social scientist has misunderstood the meaning of theorizing by which theories for the social sciences can actually be developed. The paradigm of theory development in physics is of little value if it is not understood. Theory development in physics as in any other science is the result of the logical process of retroduction by which relationships are recognized as an emendation of a point of view, whether that point of view is devised from existing theory or from the *whole cloth of relevant knowledge*.

Retroduction is the result of the imagination of the innovator and not by the mechanical process of data-mining techniques by which data-patterns are devised. Data-mining is certainly an important pursuit, although quite mechanical in nature, but it does not lead to the *creative development of theory—it is the imagination and creative insight of the innovator by which theory is developed*.

Donald Ary, Lucy Cheser Jacobs, and Asghar Razavieh. *In 1985, Ary, et al., in* Introduction to Research in Education,[24] *continue to promote the misinterpretation of how theory is developed. However, the misinterpretation is now directed at believing that when explicating a theory, that the premises; that is, axioms, must be "true." They assert:*

> We must begin with true premises in order to arrive at true conclusions. (p. 5) … The conclusions of deductive reasoning are true only if the premises on which they are based are true. (p. 6)

Unfortunately, such is not the case. The premises, axioms, must only be *presumed* to be *valid*, not *true*. For example, with respect to the three axioms cited above concerning the *Kinetic Theory of Gases*, there was no way to tell whether or not they are in fact *true*—but the scientist simply proceeded as though they were *valid*, without question. Even being able to see atoms and molecules does not make the first axiom *true*, it but *validates* the axiom and adds to the *preponderance of evidence* that in fact the axiom is *valid*.   The axiom predicted that molecules and atoms would be found, and continued experiments confirmed its *validity*. Might matter be composed of something other than "small particles"? Who knows, since we have not yet had a chance to examine all of the matter in the universe? But, at this time all physicists proceed as though all matter is so composed, since the *preponderance of evidence* indicates that they should be.

Until researchers in the social sciences understand the import of this position, no legitimate theory in education or any other social science is possible.

In addition to misunderstanding the nature of assumptions, or axioms, in the development of scientific theory, Ary, et al., also confuse the place of deduction and induction in the process of theory development. They provide the following examples:

> The difference between deductive and inductive reasoning may be seen in the following examples:
>
> A. Deductive: Every mammal has lungs.
>   All rabbits are mammals.
>   Therefore, every rabbit has lungs.
>
> B. Inductive: Every rabbit that has ever been observed has lungs.
>   Therefore, every rabbit has lungs. (pp. 6-7)

First, the example of deduction provided is that of a syllogism and not from an axiomatic theory. They are not the same. However, assuming that axiomatic deductive inferences are also included, and that the example given for induction can be appreciated, the interpretation of induction is also misleading. What has actually been demonstrated by the inductive inference is that the observations of rabbits with lungs have confirmed the deductive inference that they in fact do have lungs—the observations have *validated* the assumption with their continuing *preponderance of evidence*. Induction *validates* theory, it does not develop theory. The validation has contributed to the *preponderance of evidence* that supports the deductive inference, and, therefore, the theory.

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[24] Ary, D., Jacobs, L.C., and Razavieh, A., (1985), *Introduction to Research in Education*, Holt, Rinehart and Winston, New York.

Most telling is their lament:

> In spite of their use of the scientific approach and accumulation of a large quantity of reliable knowledge, education and the other social sciences have not attained the scientific status typical of the natural sciences. The social sciences have not been able to establish generalizations equivalent to the theories of the natural sciences in scope of explanatory power or in capability to yield precise predictions. (p. 19)

What they fail to recognize is the reason for this lack of theory development.

As Popper (Popper, 1961) points out:

> [A] science needs a point of view, and theoretical problems.[25]

And, as he confirms, the amassing of huge amounts of data does not, and cannot, amount to theory. If one were to amass the daily traffic flow at a major city intersection, one would have a large amount of data providing "reliable knowledge" about such traffic flow. However, other than gaining the knowledge that may indicate that a traffic light is required, there is nothing by which a scientific theory could be developed. This is the state of affairs in the social sciences—great amounts of knowledge have been acquired from hypothesis testing, but it is absolutely of no value for the development of an education or any other social science theory.

Rather than adhering to a process referred to as "the scientific approach," it would be more constructive to recognize that possibly even physical scientists do not follow "the scientific approach" and move to determine just what it takes to develop a theory for educologists and other social scientists. Theories in physics were not developed as the result of the "accumulation of a large quantity of reliable knowledge," they were developed as the result of the *creativity, insight and innovativeness of the researcher* to recognize emendations of existing theories or from the *whole cloth of relevant knowledge*. Until the social scientist recognizes what has to be done to be creative, the lament of Ary, et al., will continue to characterize the search for legitimate theory in the social sciences.

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[25] Popper, K., (1961), *The Logic of Scientific Discovery*, Basic Books, New York, p. 106.

A Challenge to Learning Theorists

Who is there among Learning Theorists who can take the 21 "theories of learning" cited previously and devise a theory based on first principles, basic assumptions, which will encompass every one of these 21 sets of hypotheses? That is what is needed in the social sciences—a Learning Theoretician who has the *insight and creativity* to bring under one umbrella all of the diverse hypotheses that have been propounded and validated with hundreds, even thousands, of tests. Once this is accomplished, then the validations of hypotheses will have some meaning, as they will validate the theory. As it is now all they do is contribute to the "large quantity of reliable knowledge"—but such knowledge is meaningless in terms of predicting any future outcomes and do nothing to help develop legitimate Learning Theory.

Stating hypotheses in a vacuum without their associated theory will result in confusion when trying to identify the underlying assumptions. Although Travers recognized the need to anchor hypotheses in theory, he failed to recognize what that theory had to entail. Regardless of how careful one is when preparing a hypothesis, it is almost certain that hidden or unknown assumptions have not been stated. Hypotheses must be part of some theory structure, or they are nothing more than the opinion of the researcher, even if that opinion is subsequently "validated." It is this process of hypothesis-creation that has resulted in numerous "tests" of the same subject area resulting in differing conclusions—for example, "human involvement is responsible for a substantial part of global warming," versus "humans are responsible for less than 7% of the global warming effect, and global warming will therefore occur regardless of what humans do"; or "placing girls and boys together in the same class results in better learning for all students," versus "separating boys and girls for instruction results in better learning for all students." Is it possible that with both of these hypotheses, especially the latter, that there are unstated political agendas at work that compromise the integrity of the validation?

The problem is not necessarily the tests that provide differing results, but that there is no full recognition of the underlying assumptions of the theory in which the hypothesis is stated. Theory generates hypotheses, hypotheses do not create theory nor are they themselves theory. Theory, hidden or clearly stated, produces *logically-derived, theory-based hypotheses*, or theorems in more formal theories.

The problem with the hypothetico-deductive methodology is that it does not produce theory. In education, this process has never resulted in any new theory.

To correct this problem a theory-building process that leads to legitimate theory will be presented. A proper methodology requires testing *theory-derived hypotheses* and all new applications derived from the hypotheses until the evaluations lead to a new theory that describes the problem based on first principles, "accepted and *valid assumptions*"—not "*true* premises."

Glaser and Strauss.   *As an alternative to the hypothetico-deductive methodology, Glaser and Strauss developed the* Grounded Theory *approach (Glaser, 1967). Although subsequent to the publication of their joint text Glaser and Strauss have been involved in some on-going disputes concerning the details of the approach, essentially all such approaches are flawed at the outset by grounding any theory development on acquired data.*

However, their dissatisfaction with hypothesis-driven research is well taken. The problem is that they did not recognize the underlying reason for this dissatisfaction—hypothetico-deductive methodologies or any other hypothesis-based methodology itself does not develop theory.

The *Grounded Theory* approach asserts that theory is "discovered" as the result of systematically analyzing data. As a result, this approach is very similar to, if not identical to the data mining procedures used to structure unstructured data. The response to each is the same, structuring unstructured data is certainly helpful in recognizing established patterns within systems to evaluate existing theory, but it does not produce theory.

An inductive process grounded or not, does not develop theory, whether one claims that induction was responsible for directly proposing a theory or the theory is deductively inferred from hypotheses—neither process actually resulted in theory construction.

Even trying to argue, as they do, that *Grounded Theory* is in some way associated with Peirce's abduction fails to provide a basis for theory construction. First, *abduction* is not *retroduction*. Glaser and Strauss consider *Grounded Theory* as a means for obtaining theory from data patterns—that is, data mining techniques. Theory development is a retroductive process, and not an inductive process nor an abductive process. In this case, 'abduction' as 'retroduction' is misinterpreted.

Since *Grounded Theory* is relied upon in the social sciences, we need to take a closer look at just what they say. Concerning Grounded Theory:

> Most important, it works—provides us with relevant predictions, explanations, interpretations and applications.[26] (p. 1)

This should not be surprising, since the purported theory is a direct reflection of the data. Whether or not such direct reflection provides "relevant predictions" that could not be otherwise observed is questionable. The final three criteria go directly to the fact that theory is in fact not being developed. What is being developed is akin to *Charles's Law of Gases* and *Newton's Law of Universal Gravitation*; that is, *Grounded "Theory"* is doing nothing more or less than describing what one observes concerning the interrelations of phenomena as defined by the data—"explanations, interpretations and applications."

One of the more telling representations is that it is claimed that theory is "discovered":

> The basic theme in our book is the discovery of theory from data systematically obtained from social research. (p. 2)

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[26] Glaser, B.G. and Strauss, A.L., (1967), *The Discovery of Grounded Theory: Strategies for Qualitative Research*, Chicago, Aldine Publishing Company.

It seems as though theories are out there somewhere just waiting to be "discovered" as one would discover any other empirical event or object. It should be clear that they are not.

While one must applaud both Glaser and Strauss for their dissatisfaction with theory development in the social sciences, their solution does nothing to further that end. It is pretty much irrelevant whether your hypothesis is derived *a priori* or from the ground up, hypotheses do not generate theories. To clearly discern the nature of the statements being developed, consider the following *discovery and generation of a performance-reward process* cited by Glaser and Strauss:

> In a study of organizational scientists, the analyst discovered that scientists' motivation to advance knowledge was positively associated with professional recognition for doing so. This finding suggested the theoretical inference that recognition from others maintains motivation. [Tests then followed to theoretically verify this "theoretical inference."] (p. 212)

Once again, theory seems to be something that we "discover." However, more to the point, this appears to be nothing more than an attempt to describe an event, in the same way that *Charles's Law of Gases* and *Newton's Law of Universal Gravitation* describe empirical facts. While such descriptions are certainly applicable to any number of incidents, that does not make such descriptive correlations a theory; it simply means that one has made a rather universal observation that; for example, when people are in a position of power without controls, they will abuse their position. Regardless of how many groups one analyzes to accumulate data that further confirms this observation, no theory has been developed. It may be an interesting observation, but it is not a theory.

Essentially, the scientific methodology of the social sciences has been hypothesis-driven. That is, the definitions of both induction and hypothetico-deduction theory-building methodologies are such that each relies on a hypothesis that is devoid of the foundations required of a legitimate theory.

Retroduction develops legitimate theory, whether that retroductive process results from the development of new theory from existing theory or the development of new theory from the *whole cloth of relevant knowledge*. For example, the existing theories of Set Theory, Information Theory, Graph Theory and General Systems Theory can be used to develop theory in a very analytic manner, as was done for the development of the SIGGS theory model. Alternatively, a *whole cloth* perspective of mathematics, education, chemistry, physics and the behavioral sciences develops theory by recognizing a wholeness of concepts they contain that provide a perspective that describes and predicts what is found in education systems. For example, *General Systems Theory* was developed from a *whole cloth perspective*. And, it is most likely that the challenge given previously to Learning Theorists will be resolved by one who has the insight and creativity to bring together from a *whole cloth perspective* the theories of mathematics, education, chemistry, physics and the behavioral sciences.

Further, as Popper and others have recognized, theory must be axiomatic with all of its associated safeguards. In addition, as cited above, the social sciences have attempted to produce theories that have a rigor similar to the physical sciences by introducing mathematical constructs. Although descriptive theories are possible, only logico-mathematical foundations provide the means required for general acceptance and validation. Moreover, it is essential that if logic and mathematics symbolisms are a part of a descriptive theory they are not so by mere reference, cited without substance, but the logic and mathematics must be an integral part of the theory.

This is where historical and current research in education and the social sciences generally has failed, as research continues to proceed from a position of validating hypotheses. Education research is hypothesis-driven, rather than theory-driven. While axiomatic logico-mathematical theories are far more difficult and complex than hypothesis-driven methodologies, such theories are required if educology is to move beyond a "My Theory" methodology to that of developing a consistent, comprehensive, complete, and axiomatic theory of education. And to simply assert *a priori* that formal, axiomatic, or mathematical theories cannot apply to the social sciences is a clear refutation of any "scientific process" and places such social "scientists" outside the realm of science.

To assist in bridging the gap from hypothesis-driven to axiomatic-theory-driven science, a parallel development in physics will be considered. Even in physics, which is frequently considered as being "proven" or "empirically valid," theories are considered to be acceptable for describing the physical world as a result of a "preponderance of evidence" that they produce accurate predictions of the physical world. The same will hold in the social sciences; that is, a theory is accepted because of the "preponderance of evidence" that it produces consistently valid predictions.

Following is an example of the retroductive development of a theory in physics.

> **Rock Theory**
> | | |
> | --- | --- |
> | Desired Theory: | Electrical Properties of Rocks |
> | Existing Theory: | Electrical Properties of Glass |

By conjecture: *the electrical properties of rocks are similar to the electrical properties of glass*.

Therefore, the existing *Electrical-Glass-Property Theory* is used to retroductively develop an *Electrical-Rock-Property Theory*.

This new *Rock Theory* is an emendation of the existing *Glass Theory*. As such, it brings with it the basic logic of that theory, which is comparable to other theories in physics; but, in addition it introduces new content and the resulting *Rock Theory* will contain more than what was brought to it from *Glass Theory*. *Glass Theory* provided the devising model by which *Rock Theory* is developed.

It is important to recognize that this is not an inductive process in that there is no extensive data from which "patterns" are developed that "suggest" that somehow rocks are similar to glass. To the contrary, it was the *Glass Theory* itself that was utilized to develop *Rock Theory* as the result of the *insight of the innovator* who recognized that the properties of the two mediums may be similar.

Levels of Theory Construction

There are several levels of theory construction required, especially in an axiomatic theory, before the actual desired empirical theory is obtained. These levels are discussed below, and an example from physics is presented on the pages following.

The first level consists of defining the *Basic Logics*; that is, the Sentential, Predicate, Class and Relation Calculi.

The Basic Logics provide the decisional rules by which theorems are formally derived within the theory. These provide the customary deductive logic used in physics.

Once the decisional logic is determined, then the levels of scientific inquiry must be defined. These will each require its own axioms or other structure that will be used to deduce the outcomes of the theory.

Various areas of physics will exemplify the relation between various levels of a theory. Then, the axioms or laws will be tracked to exemplify how each higher-order theory affects the lower-order theories.

In axiomatic theories, this will be accomplished by the introduction of appropriate axioms at each level.

The following tree diagram depicts various levels of theories in physics in which each lower level is dependent on the axioms or laws of the one above it, but will also introduce axioms, laws or principles that are extended from those above it.
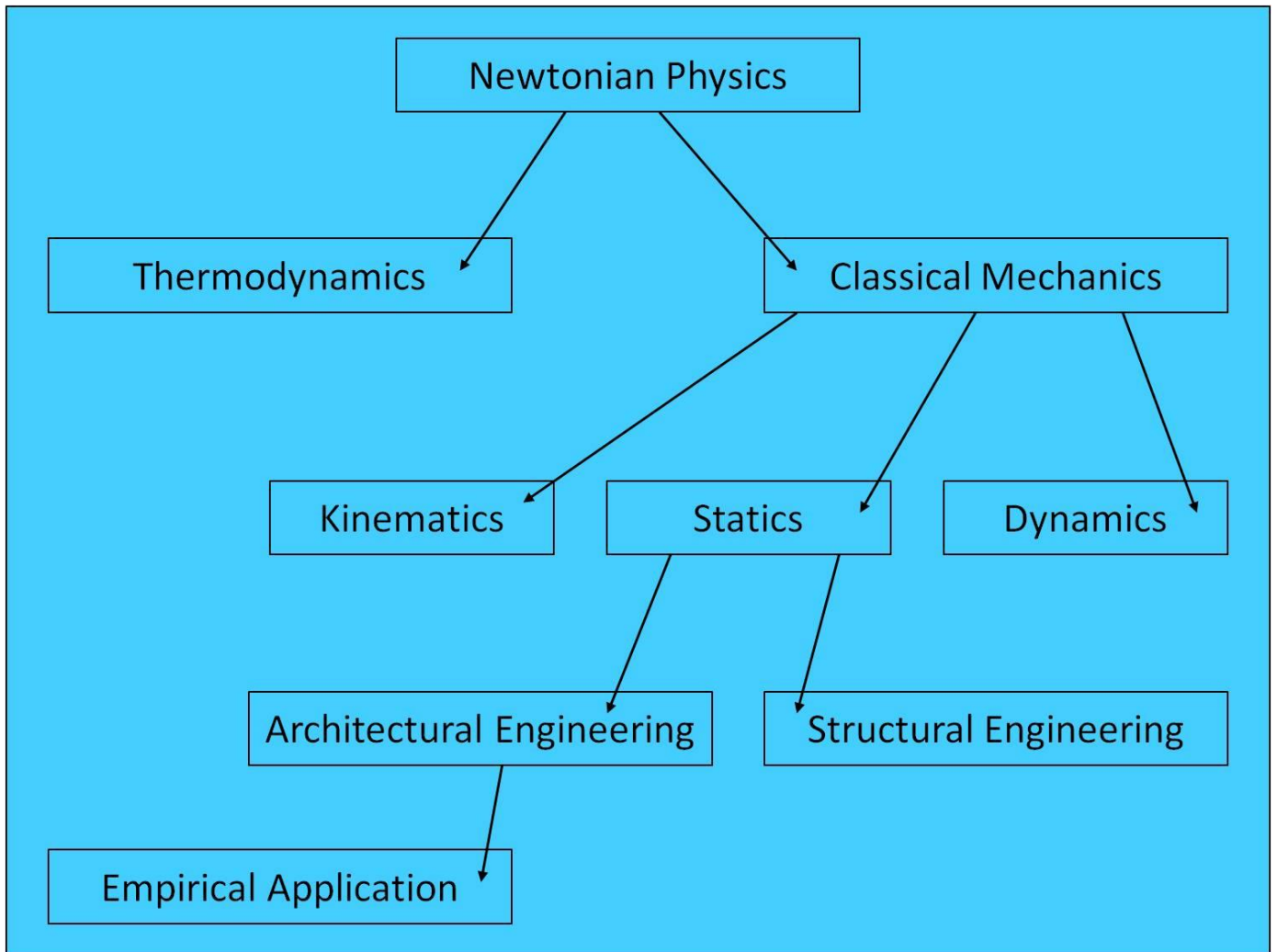
That is, recalling that the development of theory is by emendation or extension, once the initial theory has been designed by an emendation of another theory; for example, by means of a retroductive process, then the theory is constructed by extension. These extensions can be horizontal; that is, within the existing theory, or vertical; that is, by introducing sub-theories. This vertical development is shown in the following diagram. So as not to make the diagram too complex, only one area of physics is extended to the following vertical level below it; for example, "Mechanics" and then "Statics."

It is recognized that Thermodynamics has sub-theories extending below it, as do Kinematics and Dynamics. However, we will only consider the development of each of the following theories as one is extended from the one above it:

Newtonian Physics → Classical Mechanics → Statics → Architectural Engineering

Newtonian Physics

At this level we are concerned with the axioms, hypotheses, or principles that are used to explicate Newtonian Physics. In Newtonian Physics these statements are referred to as "laws" or "postulates."

Newton's Laws of Motion:

Newton's first law states that, if a body is at rest or moving at a constant speed in a straight line, it will remain at rest or keep moving in a straight line at constant speed unless a force acts upon it.

      This postulate is known as the law of inertia.

Newton's second law states that the time rate of change of the velocity or acceleration, a, is directly proportional to the force F and inversely proportional to the mass m of the body; i.e., $a = F/m$, or $F = ma$.

      From the second law, all of the basic equations of dynamics can be derived. As noted previously, this initial theory provides the main assumptions by which all extensions of the theory are obtained.

Newton's third law states that the actions of two bodies upon each other are always equal and directly opposite.

      The third law is important in statics (bodies at rest) because it permits the separation of complex structures and machines into simple units that can be analyzed individually with the least number of unknown forces.

Newton's law of gravitation is a statement that any particle of matter in the universe attracts any other with a force, F, varying directly as the product of the masses, m_1 and m_2, and a gravitational constant, G, and inversely as the square of the distance between them, R; i.e., $F = G(m_1 m_2)/R^2$.

Thermodynamics   Classical Mechanics

Continued on Next Page

Classical Mechanics: At this level we are concerned with the axioms, hypotheses, or principles that are used to explicate Classical Mechanics.

Classical Mechanics is a theory of the physics of forces acting on bodies.

The first three laws of Newtonian Physics are fundamental to Classical Mechanics and the extensions required for this theory.

In order to consider the problems relevant to Classical Mechanics, the definition of the '*position*' of a '*point particle*' is introduced. This is an extension of Newtonian Physics. With the introduction of a *point particle*, the three laws are used to develop properties relevant to Classical Mechanics. For example, properties relating to force and energy are developed from Newton's Second Law.

Classical Mechanics is subdivided into: Statics, Kinematics, and Dynamics. We will continue our vertical theory development by considering Statics.

| Kinematics | Statics | Dynamics |
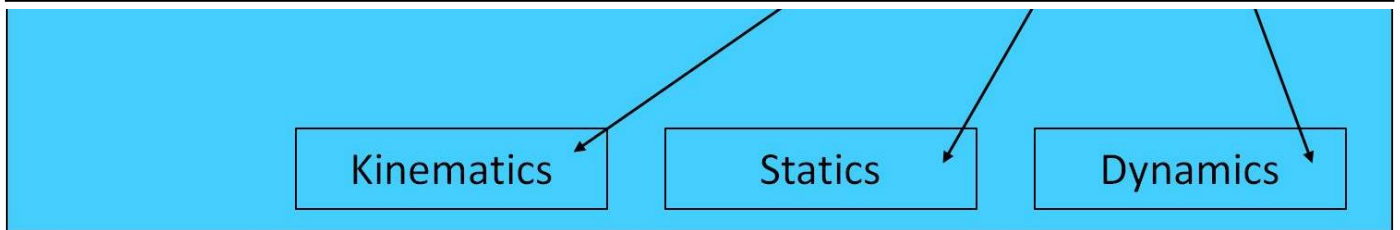
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Statics: At this level we are concerned with the axioms, hypotheses, or principles that are used to explicate Statics.

Statics is concerned with physical systems that are in static equilibrium. When in static equilibrium, the system is either at rest or moving at constant speed. By Newton's Second Law, this situation implies that the net force and net torque on every subsystem is zero. From this constraint, and the properties developed in Classical Mechanics, such quantities as stress or pressure can be derived.

1. When a wire is pulled tight by a force, F, the **stress**, σ, is defined to be the force per unit area of the wire: $\sigma = F / A$. The amount the wire stretches is called strain.

2. Failure occurs when the load exceeds a critical value for the material; the tensile strength multiplied by the cross-sectional area of the wire, $F_c = \sigma_t A$.

The theory has now been extended to include properties required for the Theory of Statics. From here, specific properties will be required for specific areas of application as shown by the next level.

Although not considered, the following definitions are provided:

Kinematics: Kinematics is the branch of mechanics concerned with the motions of objects without being concerned with the forces that cause the motion.

Dynamics: Dynamics is the branch of mechanics that is concerned with the effects of forces on the motion of objects.

| Architectural Engineering | Structural Engineering |

Continued on Next Page

Architectural Engineering: At this level we are concerned with the axioms, hypotheses, or principles that are used to explicate Architectural Engineering as an application of the Physics Theory of Statics.

For example, at this level the theory extension will be with respect to specific physical problems of concern to architectural engineers. For example, axioms, hypotheses, or principles related to the analysis of architectural structures to preclude structural failures, construction defects, expansive soils, explosions, fires, storms/hail, tornadoes, vehicular impacts and water leaks.

The theory at this level will then be used to analyze specific empirical instances.

Empirical Application

Axiomatic Logics for *ATIS*

The Argument for a Symbolic Logic

Elizabeth Steiner, in her book *Methodology of Theory Building*[27], asserts:

> One must understand the many forms (kinds) of theory if one is not to apply the wrong art, i.e., if one is not to criticize or construct theory erroneously.

This same word of caution needs to be applied to the choice of logic that underlies the development of theory. The logic of a theory provides the means by which validity of statements of the theory can be "proved" as "true," and provides the means by which valid statements of the theory are derived.

For a scientific theory, normally a *symbolic logic*; that is, *formal logic*, is desired as such provides a means to obtain rigorous proofs for the validity of statements.

The logic required for *ATIS* will be an adaptation of the *Sentential Calculus* and *Predicate Calculus* that is normally used for mathematics and the mathematical sciences. While both calculi are concerned with analyzing statements based only on the form of the statements, they differ in terms of the types of statements analyzed. The *Sentential Calculus* is concerned with the form of the aggregate statement with no concern of what is contained within the statement. The *Predicate Calculus* is concerned with the logic of predicates; that is, statements and their constituent parts, as related to quantifiers—normally the universal and existential quantifiers, although others will be required for the logic of *ATIS*.

The advantage of a symbolic logic is that proofs are dependent only on the form of the statements, and not on their content. The advantage is that while it may take great insight to discover a theorem, once discovered it can be checked very systematically. The emphasis for theory development, however, is that the theoretician must continue to rely on intuition as the primary means of theory development, and the rigors of the basic logic are but a tool to assist in this development.

Steiner defines 'intuition' as a "non-inferential form of reasoning. It is a direct intellectual observation of the essence of what is given in experience."[28]

As will be discussed later, the System Logic schemas will be presented in two forms: Those that are derived directly from the axioms and should, therefore, be considered directly descriptive of the system, and those that are "theory construction axioms" and are, therefore, to be evaluated through intuition or other analytic tools before being considered part of the theory. The definition of 'intuition' by Charles Sanders Peirce addresses this desired theory-building method very directly when he states:

> Intuition is the regarding of the abstract in a concrete form, by the realistic hypostatization of relations.[29]

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[27] *Methodology of Theory Building*, Elizabeth Steiner, Indiana University, Educology Research Associates, Sydney, 1988.

[28] *Methodology*, *Ibid*., p. 93.

[29] *Collected Papers of Charles Sanders Peirce*, Volume 1, *Principles of Philosophy*, (Editors) Charles Hartshorne and Paul Weiss, The Belknap Press of Harvard University Press, Cambridge (1960), §1.383, p. 203.

While a schema can checked by following well-defined steps, a pragmatic logic must guide the development and acceptance of the theory. The need for a pragmatic logic is especially relevant for *ATIS System Construction Theorems* (*SCTs*) that are an integral part of the theory explication. The far-reaching consequences of the introduction of this theory-development methodology is not elsewhere discussed in the literature, as far as this researcher has been able to determine, and will be only referenced herein since there may be important proprietary consequences resulting from its usage. Essentially, the value of such theorem schemas will depend on the rules of construction that are defined for their usage. However, they will be further considered in a later section, to as great a degree as possible, in the section entitled *Significance of SCTs*.

While 'formal logic' is frequently assumed, we will state precisely what is meant by such logic. For our definition, 'symbol', 'language', 'formation', and 'transformation' will be taken as primitive terms. Then, *formal logic* is defined as follows, where "$=_{df}$" is read "*is defined as*":

Formal Logic $=_{df}$ A language that contains—

(1) Symbols,

(2) Well-formed formulas derived from the symbols as determined by formation rules,

(3) Axioms that are selected well-formed formulas, and

(4) Transformation rules, normally consisting of only one, Modus Ponens.[30]

The *Predicate Calculus* can be either a first-order or higher-order logic.

In *first-order logic*, quantification covers only individual elements (components) of a specific type or class; that is, only elements of a well-defined set (class) are considered. First-order logic results in verifying properties of a class or subclass of elements.

In *second-order logic*, quantification covers predicates. Second-order logic results in verifying properties of a class or subclass of predicates.

In higher-order logic, quantification covers predicate formulas. Higher-order logic results in verifying properties of a class or subclass of predicate formulas.

Whereas the *Sentential* and *Predicate Calculi* provide the logical foundation of the empirical sciences, such application must be done with care when extending that application to *ATIS*. In fact, however, the logic required for *ATIS* is less complex than that required for mathematics and the mathematical sciences, at least initially. The reason is that mathematics and the mathematical sciences must consider distinctions between "x's" that represent "unknown" and "variable" elements. The "unknown" uses are referred to as the "free" occurrences of x, and the "variable" uses are referred to as the "bound" occurrences of x. In *ATIS*, only "bound" occurrences of x will be required.

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[30] It is noted that some treatments of a formal logic will also include Generalization as a transformation rule; however, in our logic Generalization is obtained as a theorem.

For this reason, many of the problems encountered by mathematicians relating to the *Predicate Calculus* will not be a problem in the logical analyses of *ATIS*. The reason is that, as noted above, *ATIS* does not consider any statement with free occurrences of x; that is, there are no "unknowns." As will be seen, statements with "unknowns" in *ATIS* are non-sense. For *ATIS* all uses of x are bound; that is, they are variables.

In *ATIS*, problems are not being solved in which an unknown is being sought, but what is being sought are the system relations that are true for all described components of a system. The problem with seeking unknowns in the type of statements that are being considered is that it is difficult, if not impossible, to assign any proper meaning to such statements.

For example, the following is a bound occurrence of x:

For x $\in$ $\mathbb{S}$,

$\forall x(I_p^{\uparrow}(x) \supset {}_{\mathbb{S}}\mathscr{F}^{\downarrow}(x))$; that is, "If input increases, then filtration decreases."

However, '$I_p^{\uparrow}(x)$' may or may not make sense when x is an element of just any unknown system, or even within a known system. That is, let '$I_p^{\uparrow}(x)$' be a translation of "x is the increasing input of the toput subsystem." While this English sentence is grammatically correct and has a recognizable meaning, its meaning within *ATIS* is highly suspect, since the x is now an unknown, or simply fanciful. Even if x can be construed as the input of a toput subsystem, x cannot be construed as "increasing" since it is but a single component. Or if it can be construed as increasing, then there are other assumptions of which we are not informed. x in this context is considered an unknown, or is a free occurrence of x. It is a situation in which we would have to determine under what conditions and in which systems this statement would have a proper meaning. Such statements are precluded from *ATIS* analyses.

Intentional and Complex Systems

SIGGS Theory has been developed with a strong reliance on formal theory. The formal theories of concern are symbolic logic and mathematics. This report will explicate the symbolic logic that is used to explicate *ATIS*.

In order to be selective of our logic, its application must be understood. The types of systems with which we are concerned are *Intentional Complex Systems*.

*Intentional Systems***:** Intentional Systems *are ones that are goal-oriented, or that have "intended" outcomes.*

For the analyst of general systems, an *Intentional System* is one that is predictable within certain parameters; that is, its *behavior* is predictable under certain system component relations. The challenge is to determine which system component relations are predictable and what outcomes are obtained as a result of those relations.

The problem of selecting a specific logic on which to base an analysis of general systems is that such systems are *Complex Systems*.

***Complex Systems*:** Complex Systems *are systems that are defined by large numbers of components with a large number of multiple types of heterarchy connections (affect relations) that determine the behavior of the system and such behavior is distinct from the behavior of the individual system components.*

The challenge here is to develop an analysis that can actually analyze a very large number of relations with multiple types of relations.

Complex Systems: Shown above are three examples of complex systems.
The complexity is not only in terms of the people shown, each one being a complex system,
but also the environment in terms of the foliage, structures, pottery, etc.
(Photographs by Kenneth R. Thompson)

In general, it has been concluded that such systems cannot be analyzed with linear logics, such as logics founded on implication and Modus Ponens, as are the *Sentential* and *Predicate Calculi*. However, such conclusions have been founded on the beliefs that systems cannot be analyzed that have multiple relations. Such is not the case.

Yi Lin[31] has defined systems with multiple relations. It is just such systems that are required for an analysis of *ATIS*. Further, however, the assumption that the *ATIS Predicate Calculus* is linear is misplaced. By reference only, it is recognized that an APT analysis has been incorporated into the evaluation of this systems theory, an analysis that is non-linear. The significance of this analysis resulting in a *Sentential Calculus* that is non-linear will be considered at a later time. Further, of significance to an APT analysis is that an *Axiomatic Temporal Implication Logic* has been developed that may be of value to APT and its integration with *ATIS* to develop a non-linear logic.

What is required for now is a formal method to analyze general systems, a symbolic logic and mathematical logic that formally express the properties and relations of a system such as system behavior, system structure, dynamic states, morphisms, etc.

The *Sentential Calculus* is frequently defined in terms of truth tables that provide a truth-functional analysis of statements. However, since *ATIS* is defined as an *Axiomatic Theory of Intentional Systems*, we will approach both the *Sentential Calculus* and the *Predicate Calculus* as axiomatic theories. Such an approach lends itself to clear statements of theorems and proofs. Further, such axiomatic logics are required since truth-table logics cannot address statements in general, and the complex statements of *ATIS*.

Before presenting the axioms of the theory, a brief overview will help to transition from the truth table approach to the axiomatic approach. For example, consider the Axiomatic Temporal Implication Logic shown on the next page.

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[31] Lin, Yi (1999). *General Systems Theory: A Mathematical Approach*. Kluwer Academic/Plenum Publishers, NY.

Axiomatic Temporal Implication Logic

Axiomatic Temporal Implication Logic

Temporal Implication Logic has been developed to address the logic with respect to empirical systems that have a time set and, therefore, a sequence of events. The types of relations that are of concern in this logic are those where one event precedes another in time, and the first is considered to imply the other. For example, the situation where feedin precedes feedout and there is a relation between the two that we wish to represent by an implication would fall within this classification.

Using conventional logic, paradoxes will arise whereby equivalences will result in the conclusion implying the premise, an empirical impossibility since the conclusion is subsequent in time to the premise. *Temporal Implication Logic* is designed to constructively handle temporal parameters of implication. To distinguish *Temporal Implication Logic* from the implication of the Sentential and Predicate Calculi, a distinctive symbol will be used. Whereas 'implication' for the Sentential and Predicate Calculi is normally designated as '$\supset$', *Temporal Implication*, TI, will be designated by '$\sqsupset$'.

The problem with TI is that equivalences are not valid when either predicate of the TI is negated. All other logical operations and equivalences hold. The following *Axiomatic Temporal Implication Logic* provides the logic required to formally prove theorems in an empirical theory where temporal implications occur. For this logic, the operation for negation is not allowed, while all other operations can be defined in terms of '$\sqsupset$' and '$\wedge$', which are the two basic undefined operations.

For the following axioms, $\mathcal{F}$, $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$ are statements, and x is a variable; i.e., a bound occurrence of x.

TI-A.1. $\mathcal{P} \sqsupset \mathcal{P}\mathcal{P}$

TI-A.2. $\mathcal{P}\,\mathcal{Q} \sqsupset \mathcal{P}$

TI-A.3. $(\mathcal{P} \sqsupset \mathcal{R}) \sqsupset (\mathcal{P} \sqsupset (\mathcal{Q} \sqsupset \mathcal{R}))$

T-A.4. $\forall x(\mathcal{P} \sqsupset \mathcal{Q}) \sqsupset (\forall x\mathcal{P} \sqsupset (\forall x\mathcal{Q})$

TI-A.5. $\mathcal{P} \sqsupset \forall x\mathcal{P}$, if there are no free occurrences of x in $\mathcal{P}$; i.e., no unknowns.

TI-A.6. $\forall x\mathcal{F}(x,y) \sqsupset \mathcal{F}(y,y)$

For the following axioms, $\mathcal{F}$, $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$ are statements, and x is a variable; i.e., a bound occurrence of x. The distinction between this axiom set and that of the logic of the Sentential and Predicate Calculi is that the following axiom has been removed:

$$(\mathcal{P} \sqsupset \mathcal{Q}) \sqsupset (\sim(\mathcal{Q}\mathcal{R}) \sqsupset \sim(\mathcal{R}\mathcal{P}))$$

In place of the above axiom, the following has been used:

TI-A.3. $(\mathcal{P} \sqsupset \mathcal{R}) \sqsupset (\mathcal{P} \sqsupset (\mathcal{Q} \sqsupset \mathcal{R}))$

This replacement effectively precludes $\sim\mathcal{Q} \sqsupset \sim\mathcal{P}$ as a logical equivalence of $\mathcal{P} \sqsupset \mathcal{Q}$. It also precludes numerous other equivalences in which negation of statements occur.

Following are the definitions of '$\vee$' and '$\equiv$'. The exclusive "or," '$\underline{\vee}$', cannot be defined within this *Temporal Implication Logic*.

$\mathcal{P} \vee \mathcal{Q} =_{df} (\mathcal{P} \sqsupset \mathcal{Q}) \sqsupset \mathcal{Q}$ $\mathcal{P} \equiv \mathcal{Q} =_{df} (\mathcal{P} \sqsupset \mathcal{Q}) \wedge (\mathcal{Q} \sqsupset \mathcal{P})$

This logic is designed specifically to address the problems relating to temporal implications as distinct from the standard logic that does not address this issue.

Symbolic Logic

Symbolic logic is a tool designed for scientific reasoning. In particular, it is a tool designed for *ATIS* reasoning, and also for educology reasoning; such reasoning required for a proper analysis of an *Education Systems Theory* (*EST*). It is by an interpretation of *ATIS* that educology is explicated, and by which an *EST* is retroduced.

It is through the use of symbolic logic that these theories are made precise and explicated. The main reason for using symbolic logic is that it is a means of obtaining precise definitions for the logical consequence of one statement from another. The main advantage of a formal logic is in being able to prove statements about a theory, and only minimally in being able to determine conclusions about the theory. Intuitive arguments are the more reliable source for obtaining answers, while formal arguments are required for proving those answers. For an empirical theory, like *EST*, answers are obtained by direct observation that have been predicted by the formal development of the theory and the intuitive arguments that such predictions are valid. That is, especially for the Theory-Construction Schemas, intuitive arguments are essential for guiding the application of the formal arguments.

One of the greatest advantages of a formal logic is that it provides a precise definition for determining when one statement is a logical consequence of another. When one comprehends the power of this advantage, then the fruitfulness of the predictive logic will be realized.

The objective of constructing a formal logic is that it will provide the precise criterion by which instances of *ATIS* reasoning will be determined as being correct. With this correct reasoning, one can confidently provide the predictions determined by the theory.

The logical consequence of one statement from another is obtained by a sequence of well-defined statements such that each statement is known to be valid; that is, is an axiom, is an assumption or is derived from previous statements of the sequence according to specific rules of inference.

Valid statements are only those that are axioms or are derived only from axioms.

Rules of inference are restricted to Modus Ponens.

The *ATIS* Sentential Calculus

The *ATIS Sentential Calculus* is a theory of statement formulas in which the statements are translations of sentences within *ATIS*. For *ATIS*, a statement is a declarative sentence that relates exclusively to system components, relations or properties of *ATIS*. While the *Sentential Calculus* herein considered may be equivalent to that used for mathematics and the mathematical sciences, it is important to note that the extended logic herein considered is that developed specifically for *ATIS*, and is not intended to be a logic generally utilized by mathematicians, although it may be applicable.

Statements will be expressed by capital letters; e.g., "𝒫," "𝒬," etc., and are translations of their English sentences "A" and "B," respectively. All statement functions of the theory are derived from only two undefined functions: '∧' and '~', which are read "and" and "not," respectively. [**NOTE:** Other symbols than the ones shown may be used. For example, at times the symbol '·' is used in place of '∧'.]

Therefore, '𝒫 ∧ 𝒬' is read "𝒫 and 𝒬," and is a translation of the English sentence "A and B"; and '~𝒫' is read "not 𝒫," and is a translation of the negation of the English sentence "A". While we will read '~𝒫' as "not 𝒫," the English sentence may take several forms depending on what is required to assert the negation of "A."

A statement formula is a string of statements combined with ∧ and ~.

'∧' and '~' are the first two functions of the *Sentential Calculus*:

(1)   𝒫 ∧ 𝒬

(2)   ~ 𝒫

While these functions are undefined, they will be interpreted as having "truth values" ("validity values") defined by the following "truth-value tables" ("validity-value tables"). While the values are commonly thought of as "True" or "False," in fact they are but assigned values with no relation to "truth." Instead, to further emphasize their application to empirical theories, they will be interpreted as "valid" and "not-valid". This will be emphasized in the following tables by using '⊤' for "valid" and '⊥' for "not-valid." The "validity table" then simply presents the four possible combinations of '⊤' and '⊥' in the first table and the two possible combinations in the second.

Table 1: Validity table for the operation '∧'

𝒫	𝒬	𝒫	∧	𝒬
⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊥	⊥
⊥	⊤	⊥	⊥	⊤
⊥	⊥	⊥	⊥	⊥

Table 2: Validity table for the operation '~'

𝒫	~𝒫
⊤	⊥
⊥	⊤

As demonstrated in Tables 1 and 2, the operation '∧' takes the value 'T̄' only when both P and Q are T̄; and the operation '~' takes the value that is the alternative to P.

By convention, '$P \wedge Q$' may be, and normally is, written as 'PQ'.

'$P \vee Q$' ("P or Q"—inclusive "or"; i.e., and/or), '$P \veebar Q$' ("P or Q"—exclusive "or"; i.e., not both), '$P \supset Q$' ("P implies Q" or "If P then Q"), and '$P \equiv Q$' ("P if and only if Q" or "P is equivalent to Q") are defined as follows:

(3) $P \vee Q =_{df} \sim(\sim P \sim Q)$

(4) $P \veebar Q =_{df} \sim(\sim P \sim Q) \wedge \sim(PQ)$

(5) $P \supset Q =_{df} \sim(P \sim Q)$

(6) $P \equiv Q =_{df} \sim(P \sim Q) \wedge \sim(\sim PQ) =_{df} (P \supset Q)(Q \supset P)$

These six functions are the ones by which the *Sentential Calculus* is explicated.

Since implication, $\supset$, will be a very important function of the *ATIS Sentential Calculus*, its interpretation will be further considered. The function '$P \supset Q$' may be read in any one of the following ways, all of which are equivalent:

Q is a necessary condition for P,

P is a sufficient condition for Q,

Q if P,

P only if Q,

P implies Q, and

If P then Q.

Consider a list of statements, P_1, P_2, …, P_n. Combine these statements by the use of '∧' and '~' in any manner desired, and call the result 'Γ'. As a result of this construction of Γ, Γ will be called a *statement formula*. A statement formula that is written using only '∧' and '~' will be defined as being in "standard form." The purpose of the *Sentential Calculus* is to determine when a statement formula is valid, and validity will be determined when the statement formula is "valid" pursuant to the validity-tables, or as a result of an axiomatic analysis for the *Sentential Calculus*.

Validity tables can assist in determining when a statement formula is valid regardless of the meaning of the statements that make up the formula. That is, in general, if the validity of the statements of a formula is unknown, then the validity of the formula cannot be determined.

However, statement formula validity can be determined, regardless of the validity of the statements, when the statement formula has a certain structure. For example, the statement formula "$\sim\mathcal{P}\mathcal{P}$" is always false and $\sim(\sim\mathcal{P}\mathcal{P})$ is always true regardless of the validity of $\mathcal{P}$. Validity tables can assist in determining under what conditions a statement formula is valid in the *Sentential Calculus*. Only one example will be provided since discussions on truth table analyses can be found in many introductory texts on logic.

As this is not intended to be a formal development of the *Sentential Calculus*, only the basic functions are shown as they are derived from '∧' and '∼', and their validity tables will not be shown. The reader of this report is encouraged to take a course in formal logic that at least includes Venn Diagrams, Syllogisms, and "Truth Tables" (as commonly referred to in such courses). While these studies are a beginning for the comprehension of this report, it must be understood that the logic presented herein goes far beyond the scope of an introductory logic course.

Consider the following statement formula: $\Gamma = (\mathcal{P}\wedge\sim\mathcal{Q}) \vee [\mathcal{Q} \supset (\mathcal{P} \equiv \mathcal{Q} \wedge\sim\mathcal{P})]$.

Although operations other than '∧' and '∼' are used in this statement formula, by the preceding definitions, they could be replaced with '∧' or '∼' thus writing the statement formula in standard form as is required. This also demonstrates the need for such symbolic uses, as this statement formula in standard form would be:

$$\sim[\sim(\mathcal{P}\sim\mathcal{Q})\sim[\sim(\mathcal{Q}\sim([\sim(\mathcal{P}\sim(\mathcal{Q}\sim\mathcal{P}))\sim(\sim\mathcal{P}(\mathcal{Q}\sim\mathcal{P}))])))]].$$

In this statement formula, '$\mathcal{P}$' and '$\mathcal{Q}$' are "parameters" of the formula. The question to be answered in the *Sentential Calculus* is under what conditions is this formula valid? We proceed as shown in the following tables. First, the possible values of $\mathcal{P}$ and $\mathcal{Q}$ are entered, and the table is set up so as to have a column assigned for every statement and operation. While '∼' could be given a separate column, it is normally less confusing to assign it to its associated statement.   In these tables, '⊤' designates "valid," and '⊥' designates "not-valid." Starting with *Table 4*, the values shown in bold print are determined from the values in italics. The column designated as "5" in Table 3 under '∨' determines the values for the statement formula under the possible combinations of statement validity. By the grouping symbols, the values will be determined in the order designated in the last row.

<p style="color:red; text-align:center;">**Table 3: Assign values to "P" and "Q"**</p>

$\mathcal{P}$	$\mathcal{Q}$	$(\mathcal{P}$	$\wedge$	$\sim\mathcal{Q})$	$\vee$	$[\mathcal{Q}$	$\supset$	$(\mathcal{P}$	$\equiv$	$\mathcal{Q}$	$\wedge$	$\sim\mathcal{P})]$
⊤	⊤											
⊤	⊥											
⊥	⊤											
⊥	⊥											
		1	**2**	**1**	**5**	**1**	**4**	**1**	**3**	**1**	**2**	**1**

Table 4: Determine values of statements within the formula

P	Q	(P	∧	~Q)	∨	[Q	⊃	(P	≡	Q	∧	~P)]
T	T	T		⊥		T		T		T		⊥
T	⊥	T		T		⊥		T		⊥		⊥
⊥	T	⊥		⊥		T		⊥		T		T
⊥	⊥	⊥		T		T		⊥		⊥		T

Table 5: Determine the values of the innermost operations

P	Q	(P	∧	~Q)	∨	[Q	⊃	(P	≡	Q	∧	~P)]
T	T	T	⊥	⊥		T		T		T	⊥	⊥
T	⊥	T	T	T		⊥		T		⊥	⊥	⊥
⊥	T	⊥	⊥	⊥		T		⊥		T	T	T
⊥	⊥	⊥	⊥	T		⊥		⊥		⊥	⊥	T

Table 6: Determine the values of the second-level operation

P	Q	(P	∧	~Q)	∨	[Q	⊃	(P	≡	Q	∧	~P)]
T	T	T	⊥	⊥		T		T	⊥	T	⊥	⊥
T	⊥	T	T	T		⊥		T	⊥	⊥	⊥	⊥
⊥	T	⊥	⊥	⊥		T		⊥	⊥	T	T	T
⊥	⊥	⊥	⊥	T		⊥		⊥	T	⊥	⊥	T

Table 7: Determine the values of the third-level operation

P	Q	(P	∧	~Q)	∨	[Q	⊃	(P	≡	Q	∧	~P)]
T	T	T	⊥	⊥		T	⊥	T	⊥	T	⊥	⊥
T	⊥	T	T	T		⊥	T	T	⊥	⊥	⊥	⊥
⊥	T	⊥	⊥	⊥		T	⊥	⊥	⊥	T	T	T
⊥	⊥	⊥	⊥	T		⊥	T	⊥	⊥	⊥	⊥	T

Table 8: Determine the values for the statement formula

P	Q	$(P$	$\wedge$	$\sim Q)$	$\vee$	$[Q$	$\supset$	$(P$	$\equiv$	Q	$\wedge$	$\sim P)]$
T	T	T	$/$	⊥	⊥	T	$/$	T	⊥	T	⊥	⊥
T	⊥	T	**T**	T	**T**	⊥	T	T	⊥	⊥	⊥	⊥
⊥	T	⊥	$/$	⊥	⊥	T	$/$	⊥	⊥	T	T	T
⊥	⊥	⊥	$/$	T	**T**	⊥	T	⊥	⊥	⊥	⊥	T

Therefore, this statement formula is valid when P is true and Q is false, or when both are false.

While the above example of a statement formula is rather simple, the determination of its validity is somewhat tedious, and was elaborated in six steps to make that very point. While "truth tables" can be used to determine the validity of many statement formulae, they are quite cumbersome when either long statement formulae are evaluated, or more than two different parameters are part of the formula. For example, eight rows of values are required for statements containing three distinct parameters, and 16 rows are required for formulas with four distinct parameters. In general, 2^n rows are required for n distinct parameters. An even more complex evaluation arises if; for example, a "contingent" value, C, is introduced. In this case, two parameters would require nine rows, and in general 3^n rows are required for n distinct parameters.

To see the efficacy of moving from validity table analyses to axiomatic analyses, we will evaluate the statement formula: $P \supset [(P \supset Q) \supset Q]$ as shown in Table 9.

Table 9: Statement formula, P ⊃ [(P ⊃ Q) ⊃ Q], analysis

P	Q	$\sim Q$	$\supset$	$[(P$	$\supset$	$Q)]$	$\supset$	$\sim P]$
T	T	T	**T**	T	T	T	T	T
T	⊥	T	**T**	T	⊥	⊥	T	⊥
⊥	T	⊥	**T**	⊥	T	T	T	T
⊥	⊥	⊥	**T**	⊥	T	⊥	⊥	⊥

Modus Ponens

From Table 9, it is seen that the statement formula is a tautology; i.e., it takes the value "$\top$" under any value of P and Q. This statement formula is so important that it has been given the name "*Modus Ponens*." Expressed in analytic form, the statement formula is:

$$P, P \supset Q \vdash Q$$

This formula is read: "P and $P \supset Q$ yields Q." This formula means that if you are given P and $P \supset Q$, you can conclude Q.

Modus Ponens is applied when you have either proven that P and $P \supset Q$ are valid or are assumptions. The importance of Modus Ponens for our theory is that it provides the one and only logical rule for proving theorems.

Modus Talens

Another important statement formula is entitled "*Modus Talens*," and has the form $\sim Q \supset [(P \supset Q) \supset \sim P]$. The validity table for this statement formula is shown in Table 10.

Table 10: Statement formula, ~Q ⊃ [(P ⊃ Q) ⊃ ~P], analysis

P	Q	$\sim Q$	$\supset$	$[(P$	$\supset$	$Q)]$	$\supset$	$\sim P]$
$\top$	$\top$	$\perp$	$\top$	$\top$	$\top$	$\top$	$\perp$	$\perp$
$\top$	$\perp$	$\top$	$\top$	$\top$	$\perp$	$\perp$	$\top$	$\perp$
$\perp$	$\top$	$\perp$	$\top$	$\perp$	$\top$	$\top$	$\top$	$\top$
$\perp$	$\perp$	$\top$	$\top$	$\perp$	$\top$	$\perp$	$\top$	$\top$

Modus Talens is used when you have $\sim Q$ and $P \supset Q$. Given these two statement formulas, you can conclude ~P. Formally, this statement formula is: $\sim Q, P \supset Q \vdash \sim P$.

Numerous logical tautologies can be confirmed by the use of validity tables. Having done so, the results can be used without recourse to the validity tables. For example, by use of validity tables, it can be shown that '$\supset$' is a transitive operation. That is, if $P \supset Q$ and $Q \supset R$, then we can conclude that $P \supset R$. With the great number of axioms contained in *ATIS*, the transitivity operation greatly facilitates the proving of numerous theorems. Formally, this transitivity property is: $P \supset Q, Q \supset R \vdash P \supset R$.

Applying the transitivity property of '$\supset$' to the following SIGGS axioms the efficacy of such formal treatments of theories is seen once again.

Consider Axioms 144 and 150, stated as follows:

Axiom 144: If filtration decreases, then isomorphism increases.

Axiom 150:   If automorphism increases, then input increases and storeput increases and fromput decreases and feedout decreases and filtration decreases and spillage decreases and efficiency decreases.

Stated formally, these axioms are:

Axiom 144: $\mathscr{R}^{\downarrow} \supset \mathscr{I}^{\uparrow}$

Axiom 150: $\mathscr{A}^{\uparrow} \supset I_{p}^{\uparrow} \wedge S_{p}^{\uparrow} \wedge F_{p}^{\downarrow} \wedge f_{o}^{\downarrow} \wedge \mathscr{R}^{\downarrow} \wedge \mathscr{R}^{\downarrow} \wedge S_{E}^{\downarrow}$

For Axiom 150, we will select only one of the conclusions, $\mathscr{R}^{\downarrow}$, to prove.

Given: Axiom 150: $\mathscr{A}^{\uparrow} \supset \mathscr{R}^{\downarrow}$; and

                    Axiom 144: $\mathscr{R}^{\downarrow} \supset \mathscr{I}^{\uparrow}$.

              $\therefore\ \mathscr{A}^{\uparrow} \supset \mathscr{I}^{\uparrow}$

That is, from the transitivity of '$\supset$', we can obtain the theorem: $\vdash \mathscr{A}^{\uparrow} \supset \mathscr{I}^{\uparrow}$ from Axioms 150 and 144.

That is, if system automorphism increases, then isomorphism increases, and no validity tables are required to prove this theorem.

The purpose of a symbolic logic is to be able to consider the parameters of a theory without recourse to the meaning of the concepts, and the purpose of the *Sentential* and *Predicate Calculi* is to derive theorems based solely on the form of the statement formulas. This task of deriving theorems can be more easily accomplished by the use of an axiomatic approach to the *Sentential* and *Predicate Calculi*.

A note is required concerning the meaning of the symbols used above. Since they are part of a statement formula, they must be "statements." For example, the above formalization of Axiom 144 is of the statement: "If filtration decreases, then isomorphism increases." Formally, this is of the form "$\mathcal{P} \supset \mathcal{Q}$," where '$\mathcal{P}$' is "filtration decreases" and '$\mathcal{Q}$' is "isomorphism increases." However, there is more contained in these statements to actually make them "statements." They will be considered to read as follows: '$\mathcal{P}$' is "The system filtration decreases"; and '$\mathcal{Q}$' is "The system isomorphism increases," both of which are now declarative sentences about an *ATIS* system. Whenever a property is cited in an axiom, it is to be understood that the property is actually contained within a statement.

Axiomatic Sentential Calculus

Whereas validity tables are convenient for determining the validity of statement formulas, such tables cannot be generalized to all statements. To date, only an axiomatic method is known that is able to obtain validations of general statements. As a transition to the axioms required for validation of general statements, we will first consider a subset of those axioms, the validity-value axioms. These axioms will provide an excellent transition to axiomatic logic, since these axioms will produce those statements considered earlier, the statement formulae that can be validated by use of a validity table, and, therefore can be easily validated by two methods—validity tables and axioms.

In general, the axiomatic definition of valid statements is obtained by the following process: (1) Certain selected statements are called 'axioms' (and their selection may be somewhat arbitrary and may be modified to achieve certain objectives); (2) A transformation rule is selected, normally Modus Ponens (although other transformation rules are possible; for example, Generalization or Modus Talens); and (3) 'Valid statements' are those statements that are either axioms or can be derived from two or more axioms by successive applications of Modus Ponens.

It is worth mentioning again that only the form of the statements and not their meaning determines valid statements.

There are three axioms of the *Valid-Value Sentential Calculus* and one logical rule.

Let '$\mathcal{P}$', '$\mathcal{Q}$', and '$\mathcal{R}$' be statements of the theory, then—

The logical rule is Modus Ponens and the axiom schemas are:

(1) $\mathcal{P} \supset \mathcal{PP}$

(2) $\mathcal{PQ} \supset \mathcal{P}$

(3) $\mathcal{P} \supset \mathcal{Q} .\supset. \sim(\mathcal{QR}) \supset \sim(\mathcal{RP})$

There are an infinite number of statements that will comprise the valid-value axioms; however, all axioms will be of one of the above three general forms, the *axiom schemas*. Further, all theorems of the *Valid-Value Sentential Calculus* can be derived from these three axioms and Modus Ponens.

A theorem will take the form: $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n \vdash \mathcal{Q}$, where the $\mathcal{P}$'s are statements and $\mathcal{Q}$ is an axiom, or $\mathcal{Q}$ is one of the $\mathcal{P}$'s, or $\mathcal{Q}$ is derived from the $\mathcal{P}$'s by repeated applications of Modus Ponens.

'$\vdash$' is read "yield", or in the case when we have only '$\vdash \mathcal{Q}$' it is read "yields $\mathcal{Q}$."

The theorem $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n \vdash \mathcal{Q}$ indicates that there is a sequence of statements, $S_1, S_2, \ldots, S_m$, called the *proof* of the theorem, such that S_m is $\mathcal{Q}$ and for each S_i, either:

(1) S_i is an axiom,

(2) S_i is a $\mathcal{P}$; i.e., an assumption,

(3) S_i is the same as some earlier S_j, or

(4) S_i is derived from two earlier S's by Modus Ponens.

The sequence $S_1, S_2, …, S_m$ is a proof within the Symbolic Logic so that Q is logically derived from the assumptions $\mathcal{P}_1, \mathcal{P}_2, …, \mathcal{P}_n$.

The following theorems are readily provable concerning $\vdash$:

Theorem. If $\mathcal{P}_1, …, \mathcal{P}_n \vdash Q$, then $\mathcal{P}_1, …, \mathcal{P}_n, \mathcal{R}_1, …, \mathcal{R}_m \vdash Q$.

Proof: Let $S_1, …, S_s$ be the proof of $\mathcal{P}_1, …, \mathcal{P}_n \vdash Q$ where S_s is Q. Clearly that same sequence will yield Q regardless of any additional assumptions.

Theorem. If $\mathcal{P}_1, …, \mathcal{P}_n \vdash Q_1$ and $Q_1, …, Q_m \vdash \mathcal{R}$, then $\mathcal{P}_1, …, \mathcal{P}_n, Q_2, …, Q_m \vdash \mathcal{R}$.

Theorem. If $\mathcal{P}_1, …, \mathcal{P}_n \vdash Q_1, \mathcal{R}_1, …, \mathcal{R}_m \vdash Q_2$, and $Q_1, …, Q_q \vdash S$, then

$$\mathcal{P}_1, …, \mathcal{P}_n, \mathcal{R}_1, …, \mathcal{R}_m, Q_3, …, Q_q \vdash S.$$

Theorem. If $\vdash Q_1$ and $Q_1, …, Q_m \vdash \mathcal{R}$, then $Q_2, …, Q_m \vdash \mathcal{R}$.

Theorem. If $\vdash Q_1, \vdash Q_2, …, \vdash Q_m$ and $Q_1, …, Q_m \vdash \mathcal{R}$, then $\vdash \mathcal{R}$.

Since our main concern is to provide the means to explicate *ATIS*, the *Sentential Calculus* will not be further explicated. The following *List of Logical Schemas* is provided to facilitate the explication of the theories. This list is not exhaustive, but does represent those schemas that lend themselves to a fruitful explication of the theories. Following this list, the schemas will be used to demonstrate the value of such a symbolic logic by providing proofs of theorems. It is noted that technically these schemas are not actually part of the *Sentential Calculus* but are part of the metatheory, the *Meta-Sentential Calculus*. They are statements about the calculus that define the form or schemas that the theorems of the theory actually take.

List of Logical Schemas

The following list of the logical schemas is provided to facilitate the proof of theorems. The proof of various theorems for *ATIS* will be presented in a separate report.

The "*System Construction Theorems*" (*SCTs*), derived directly from the axioms of the *Sentential Calculus* and the intuitive creativity of a researcher or interpreter, provide a means of developing the connectedness of a system or of determining predictive outcomes. These should prove important in developing the system topology. The significance and use of *SCTs* will be clarified before presenting the logical schemas.

Significance of SCTs

System Construction Theorems (SCTs) provide the means to develop, enhance or further the explication of a theory. The significance is that they provide additional statements than what are found in the assumptions. Since they are statements of the theory, however, they are valid statements, they are not just any statements whimsically selected.

They may, however, be statements that are intuitively derived and thereby declared to be valid statements of the theory. As an initial example; however, consider the case where the derived statement is an axiom. As an axiom, it is a valid statement of the theory.

Consider Logical Schema 3: $\mathcal{P} \supset \mathcal{R} \vdash \mathcal{P} \supset (\mathcal{Q} \supset \mathcal{R})$.

Let $\mathcal{Q}$ be Axiom 105 of SIGGS: "If centrality increases, then toput decreases."

Then, regardless of what $\mathcal{P}$ and $\mathcal{R}$ represent, the following is valid:

$\mathcal{P} \supset \mathcal{R} \vdash \mathcal{P} \supset$ ("If centrality increases, then toput decreases" [Axiom 105] $\supset \mathcal{R}$),
      where $\mathcal{P}$ and $\mathcal{R}$ are statements of the theory and $\mathcal{P} \supset \mathcal{R}$ is assumed to be valid.

For example:

Let $\mathcal{P}$ be the statement: "System complete connectivity increases"; and

Let $\mathcal{R}$ be the statement: "System feedin increases."

Then, $\mathcal{P} \supset \mathcal{R}$ is a statement of Axiom 100; and, therefore $\mathcal{P} \supset \mathcal{R}$ is valid.

Then, from our theorem we have:

> "System complete connectivity increases" ⊃ "System feedin increases" ⊢
>
> > "System complete connectivity increases" ⊃ ("If centrality increases, then toput decreases" ⊃ "System feedin increases").

The conclusion of this statement is equivalent to the following:

> > "System complete connectivity increases" ⊃ ("centrality decreases or toput decreases" ⊃ "System feedin increases").

It is probably clear that this is a non-obvious theorem; hence the value of the formal logic is established. But, what does it tell us?

This theorem provides a means to control a system. If the target system has complete connectivity increasing and system feedin increasing then the assumption of the theorem is satisfied. Now, assume that the target system is a terrorist system and that it is desired to decrease the complete connectivity. One way to accomplish this is to decrease toput and feedin. By decreasing toput and feedin under these conditions, system complete connectivity will decrease. Further, decreasing toput decreases feedin. Therefore, only one factor, toput, has to be controlled in order to achieve the objective of decreasing complete connectivity.

This analysis demonstrates several points. First, there are numerous non-obvious theorems that can be derived from a logical axiomatic analysis of the theory. Second, some of the outcomes, as with the above theorem, are counter-intuitive. In this case, the measure of complete connectivity is dependent on the potential complexity of the system, such complexity being degraded when toput is reduced. Third, the *SCTs* provide a fruitful means to analyze a system, but may require the intuitive skill of the analyst. On the other hand, where the logic is required for applications similar to SimEd, by defining certain "replacement" or "substitution" rules that will allow for selection of various properties or newly acquired data such logic can be programmed. These rules will probably have to be developed by an analyst who has a grasp of the pragmatic content of the theory.

Logical Schemas

SCTS: "System Construction Theorem Schema".

Logical Schema 0: $\quad\quad\quad P \supset Q, Q \supset R \vdash P \supset R$ $\quad\quad\quad\quad\quad$ (Transitive Property of $\supset$)

Logical Schema 1: $\quad\quad\quad P \supset Q, R \supset Q \vdash P \vee R \supset Q$

Logical Schema 2: $\quad\quad\quad P \supset Q, R \supset S \vdash PR \supset QS$

Logical Schema 3: $\quad\quad\quad P \supset R \vdash P \supset (Q \supset R)$ $\quad\quad\quad\quad\quad$ (SCTS)

Logical Schema 4: $\quad\quad\quad P \supset Q, P \supset R \vdash P \supset QR$

Logical Schema 5: $\quad\quad\quad \vdash Q \supset P .\equiv. {\sim}P \supset {\sim}Q$

Logical Schema 6: $\quad\quad\quad$ **If** $\;P \supset Q$**, then** $P \vdash Q$**; and If** $\;P \vdash Q$**, then** $P \supset Q :\equiv:$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad P \vdash Q .\equiv. \vdash P \supset Q$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ "$P \vdash Q . \supset. P \supset Q$" is the ***Deduction Theorem***.

Logical Schema 7: $\quad\quad\quad \vdash {\sim}({\sim}PP)$

Logical Schema 8: $\quad\quad\quad \vdash {\sim}{\sim} P .\equiv. P$

Logical Schema 9: $\quad\quad\quad \vdash {\sim}P \vee P$

Logical Schema 10: $\quad\quad\quad P \vdash Q \supset PQ$ $\quad\quad\quad\quad\quad\quad\quad$ (SCTS)

Logical Schema 11: $\quad\quad\quad {\sim}(QR) \vdash R \supset {\sim}Q$

Logical Schema 12: $\quad\quad\quad P \supset Q \vdash PR \supset QR$ $\quad\quad\quad\quad\quad$ (SCTS)

Logical Schema 13: $\quad\quad\quad R \supset S \vdash PR \supset PS$ $\quad\quad\quad\quad\quad$ (SCTS)

Logical Schema 14: $\quad\quad\quad PQ \supset P \vdash P \supset (Q \supset R)$ $\quad\quad\quad$ (SCTS)

Logical Schema 15: $\quad\quad\quad \vdash PQ \supset R .\equiv. P \supset (Q \supset R)$

Logical Schema 16: $\quad\quad\quad P \supset {\sim}Q \vdash P \supset (Q \supset R)$ $\quad\quad\quad$ (SCTS)

Logical Schema 17: $\quad\quad\quad P \supset {\sim}R \vdash P \supset {\sim}(QR)$ $\quad\quad\quad$ (SCTS)

Logical Schema 18: $\quad\quad\quad P \supset Q, P \supset {\sim}R \vdash P \supset {\sim}(Q \supset R)$

Logical Schema 19: $\quad\quad\quad P, P \supset Q \vdash Q$ $\quad\quad\quad\quad\quad\quad\quad$ (Modus Ponens)

Logical Schema 20: $\quad\quad\quad {\sim}Q, P \supset Q \vdash {\sim}P$ $\quad\quad\quad\quad\quad\quad$ (Modus Talens)

The *ATIS Predicate Calculus*

While the *Sentential Calculus* has been well presented so as to demonstrate the usefulness of a formal logic, the *ATIS Predicate Calculus* will be only briefly discussed with what is required to understand its application to the analysis of the target theories. Unlike the *Sentential Calculus*, however, it is important to note that this *Predicate Calculus* is distinctly different from that required for mathematics or the mathematical sciences. Without going into any great discussion, the reason is that for *ATIS* only bound occurrences of x are considered since free occurrences do not have any apparent meaning within *ATIS*.

It was previously stated that the difference between the *Sentential* and *Predicate Calculi* was that the *Sentential Calculus* is concerned with the form of the aggregate statement with no concern of what is contained within the statement, whereas the *Predicate Calculus* is concerned with the logic of predicates; that is, statements and their constituent parts, as related to quantifiers—normally the universal and existential quantifiers. This extension will now be considered.

To make the transition from the *Predicate Calculus* required for the traditional mathematical sciences and that required for the mathematical science of *ATIS*, we will first consider the predicate notation. The predicate notation will take the form of a function; e.g., $\mathcal{P}(x)$, where 'x' is an "unknown." If we can prove that $\mathcal{P}(x)$ is valid for the unknown 'x', then we have $\vdash \mathcal{P}(x)$. If we have $\vdash \mathcal{P}(x)$ then we can replace 'x' with a variable and will conclude: $\vdash \forall x \mathcal{P}(x)$. For *ATIS*, it is assumed that all predicates are bound, and, therefore, all occurrences of x are variables and the validity-value of all predicate functions can be determined. Therefore, with respect to any occurrence of x, the task is to assert $\vdash \forall x \mathcal{P}(x)$ and determine if a proof exists.

Since Alonzo Church, in 1936, proved that there is no decision procedure for the *Predicate Calculus*, then the only affirmative conclusion that is possible concerning $\vdash \forall x P(x)$, with respect to the *Predicate Calculus*, is that it is valid. If no such affirmative conclusion can be found, then nothing more can be said concerning the validity of the statement. Further, the conclusion is even stronger. Church proved that there is no decision procedure regardless of what axioms are considered.

This is great news for the logician and for any researcher or analyst who is attempting to evaluate *ATIS* or an *EST* (*Education Systems Theory*). What Church has proved is that there will always be a need for the researcher and analyst, since the *Predicate Calculus*, and the *ATIS Predicate Calculus*, in particular, has no decision procedure, and, therefore, cannot be fully programmed. It is not asserted that the *ATIS Predicate Calculus* cannot be partially programmed, because it can be, but it cannot be completely programmed. The part that can be programmed, as seen below, is that part that results from the axioms that define the *ATIS Sentential Calculus*.

This point is worth elaborating. This researcher has been attempting to define the scope of this theory that is programmable, since such programs will clearly make the theory and any of its proprietary software products more appealing to users. As reflected by the extensive list of theorems that can be derived from the *ATIS Sentential Calculus*, numbering in the tens-of-thousands, and the numerous *Theorem Schemas* cited previously, it is seen that a very fruitful analysis of a system can be obtained.

Further, this researcher has proposed that utilizing data mining technologies can even extend the value of this fruitful analysis. That is, the theory software can be used as an interpreter of the data mining structured outcomes, thus enhancing the time-sensitive results required in a terrorist environment, and possibly in an educational environment. With this technology, it is no longer required that one must wait for a pattern to be determined by the data mining, but that the theory analysis will enhance the ability of the data mining technology to recognize patterns and predicted terrorist behavior or targets much earlier than utilizing the data mining technology alone.

That said, it must also be recognized that when evaluating a specific system, the logic is only semi-decidable. That is, an analyst can affirmatively determine, within the theory, that a theorem is valid, but cannot, under any circumstances, prove that it is not-valid. The reason for this is three-fold: (1) As soon as an empirical system is recognized, the problem for the analyst reverts to considerations within the *ATIS Predicate Calculus*; (2) Church has proved that such considerations are only semi-decidable; and (3) The reason that such problems considered in an empirical system are only semi-decidable is that one never knows if all possibilities have actually been considered in the proof. Systems, especially behavioral systems, are complex. This must be recognized and recognized as something positive. That is, the researcher and analyst have some very difficult tasks confronting them.

So, is the analyst without recourse? Not at all. Creative proofs from outside the theory are possible. If a reasoned argument can be found that can be construed as part of the logic of the meta-theory, then a particular theorem can be cited as being not-valid. Once the theorem has been proved in the meta-theory as being not-valid, one is then justified by claiming that the theorem is not-valid within the theory. The significance of this is that the results of this proof can then be inserted into the theory as though it had been proved within the theory.

This researcher has previously cautioned against inserting theorems directly into a computer program that has been developed as a model of the theory. However, that precaution was with respect to the *ATIS Sentential Calculus*. The *ATIS Predicate Calculus* is an entirely different matter. Whereas the theorems of the *ATIS Sentential Calculus* can be obtained directly from the axioms and, therefore, do not warrant the arbitrary insertion of theorems, the same cannot be said for the *ATIS Predicate Calculus*. Further, there will be additional work for the researcher and analyst once the initial logic has been developed and implemented for theory model applications. There are additional analyses that can be made with respect to empirical systems. It is intended that the structural properties of a system can be recognized as the topology of the system, and that the power of mathematical topology can be modified, as the *Predicate Calculus* has been, in such a way that the power of a modified mathematical topology can be used to assist in the analysis of a system. Such analyses may also have to be performed by a researcher or analyst directly with little or no reliance on a computer program. These results also will have to be manually inserted into any computer program that has been designed for a particular system.

What this simply means is that researchers and analysts of behavioral systems will always have a job. To this researcher, that is something to look forward to.

As noted previously, due to the nature of the target theories, there will be no need to distinguish between "free" and "bound" occurrences of 'x', since, without any loss of generality, all occurrences of 'x' are considered to be bound. In view of this, we have the following axioms:

(1) $\mathcal{P} \supset \mathcal{PP}$

(2) $\mathcal{PQ} \supset \mathcal{P}$

(3) $\mathcal{P} \supset \mathcal{Q} :\vee: \sim(\mathcal{QR}) \supset \sim(\mathcal{RP})$

(4) $\forall x(\mathcal{P} \supset \mathcal{Q}) :\supset: \forall x\mathcal{P} \supset \forall x\mathcal{Q}$

(5) $\mathcal{P} \supset \forall x\mathcal{P}$

(6) $\forall x\mathcal{P}(x,y) \supset \mathcal{P}(y,y)$

It should be recognized that the first three axioms are simply taken from the *Sentential Calculus*; that is, all such resulting theorems are still valid in the *Predicate Calculus*.

As seen from the axioms, the only quantifier is the universal quantifier. The existential quantifier will be defined in terms of the universal:

$$\exists x\mathcal{P} =_{df} \sim\forall x\sim\mathcal{P}$$

From this definition, we have the following equivalences:

$\vdash \exists x\mathcal{P} \equiv \sim\forall x\sim\mathcal{P}$

$\vdash \forall x\mathcal{P} \equiv \sim\exists x\sim\mathcal{P}$

$\vdash \sim\exists x\mathcal{P} \equiv \forall \sim\mathcal{P}$

$\vdash \sim\forall x\mathcal{P} \equiv \exists x\sim\mathcal{P}$

There are special conditions for $\exists$ for which additional notations are desired. These are the conditions in which there is exactly one x for which P is valid and when there are n x's for which P is valid. These notations are as follows:

$\exists^1 x\mathcal{P}(x)$ denotes that there is only one x for which $\mathcal{P}(x)$ is valid; and

$\exists^n \mathcal{P}(x)$ denotes that there are exactly n x's for which $\mathcal{P}(x)$ is valid.

In addition to the two quantifiers, $\forall$ and $\exists$, there are two additional quantifiers, one that will be used to specify a single component and one that will specify a class of components. These quantifiers are the descriptor quantifier, ι, and the class quantifier, $\hat{w}$. '$\iota x\mathcal{P}(x)$' is read, "the x such that P(x)"; and '$\hat{w}\mathcal{P}(w)$' is read "the class of w determined by $\mathcal{P}(w)$." These are defined as follows:

$\iota x\mathcal{P}(x) =_{df} \exists^1 x\mathcal{P}(x)$; and

$\hat{w}\mathcal{P}(w) =_{df} \iota\alpha\forall w(w \in \alpha \equiv \mathcal{P}(w))$

'$\iota x\mathcal{P}(x)$' is the *name* of the unique object that makes $\mathcal{P}(x)$ valid.

The class quantifier gives a convenient means for defining a universal class and a null class. If there are no w's in $\mathcal{P}$, then $\hat{w}\mathcal{P}$ designates the universal class, $\mathcal{U}$, if $\mathcal{P}$ is valid, and the null class, Ø, if $\mathcal{P}$ is not-valid.

An important clarification needs to be made concerning the meaning of 'quantifier'. A logical quantifier designates a qualification of a class by indicating the logical quantity; that is, the specific components to which the qualification applies. '$\mathcal{P}$' or '$\mathcal{P}(x)$' is the *scope* of the quantification; that is, the scope of what is qualified. This will be a frequently used concept in the analysis of systems. The '*Logistic Qualifiers*' are those predicates that will be used to quantify a specific set. For example, *Toput* becomes *Input* as the result of quantifying *Toput* with respect to the *Logistic Qualifiers*. This system transition function is defined as follows:

$$\sigma{:}(T_\mathcal{P} \times \mathscr{L}_{i=1{:}n}(\mathcal{P}_i(w \in T_\mathcal{P})) \to I_\mathcal{P}\,) = \hat{w}_{I_\mathcal{P}}\mathcal{P}(w_{T_\mathcal{P}})$$

where 'i=1:n' designates "i varies from 1 to n," and '$\mathcal{P}_i(w \in T_\mathcal{P})$' is a qualifying statement in $\mathscr{L}$ with respect to w in $T_\mathcal{P}$.

'$\hat{w}_{I_\mathcal{P}}\mathcal{P}(w_{T_\mathcal{P}})$' designates the *Input Class* determined by the *Toput Class* qualified by the $\mathcal{P}(w)$'s in $\mathscr{L}$ that make $\mathcal{P}(w_{T_\mathcal{P}})$ valid.

An equivalent notation for $\hat{w}\mathcal{P}$ is {w | $\mathcal{P}$}, which is frequently used in mathematics.

Theory Building

In the preceding sections, the need and requirements for an axiomatic logic have been presented. In that discussion the problems relating to theory building that relies on induction, hypothetico-deductive and grounded methodologies were discussed. Now we will consider some specific concerns relating to theory building itself. What follows will be a discussion of several specific points and how to determine if theory building is actually being pursued, and if it is, what one must look for in that theory building and how to validate the theory once it is developed.

First, we will consider how to determine if the validation of a hypothesis is theoretically sound. The basic test is simply to ask the following question:

> Was the hypothesis derived from a theory that is comprehensive, consistent and complete; and, if so, is the theory axiomatic?

With respect to the requirement that the theory be axiomatic, it is simply a recognition that only axiomatic theories have been found to provide the rigorous analyses required to obtain confidence in the theory results. If an axiomatic theory cannot be obtained, then the results can always be questioned either with respect to the validation process or with respect to the "underlying assumptions" that are not stated in the theory. Descriptive and statistical-based theories can never be individually predictive and any results can always be questioned with respect to the descriptive theory, and statistical-based theories, by definition, can never be individually predictive.

Put another way, simply ask yourself:

> Was the hypothesis derived from theory? If so, what is it?

Once the theory has been established, then the next question that needs to be addressed concerns the logical basis of the theory. Most often it will be founded on a *Predicate Calculus*. If so, then there are additional questions that relate to that logic.

Any theorems that are derived from the *Predicate Calculus* are a result of the form of the theorems and not their content. The theorems of the theory that are derived directly from the basic logic are true because of their logical structure, and not at all because of their content.

In addition to theorems that are derived from the basic logic, there will be theorems that are derived from *ATIS-axioms*. Further, there will be theorems that are derived from the axioms obtained as a result of the specific empirical system being considered. Axioms and theorems from the latter two will depend upon the meaning of the terms employed within the theory or system, and not due only to their logical structure.

Class Calculus

Before considering the axioms of *ATIS* and how to develop axioms for specific systems, the axioms of the *Predicate Calculus* will be extended to include the *Class Calculus*. For this extension, a more precise and formal development of the basic logic will be presented so that a clear definition of *term*, *statement* and *formula* can be obtained.

Stratified statements determine classes. However, for *ATIS*, the initial partitioning of the system components and the definition of the system affect relations determine the stratification. Affect relations are, by definition, one class or type higher than the system components, and there is, therefore, no confusion of types.

A statement is determined by the following symbols:

$$\sim \ \wedge \ \forall \ \iota \ \hat{} \ \in \quad \text{``}variables\text{''}: x_1 \ x_2 \ \dots \ x_n \ x \ y \ \alpha \ \beta \qquad \text{``}statements\text{:''} \ \mathcal{P}$$
$$\mathcal{Q} \ \ \mathcal{R} \ \ \mathcal{P}(x) \ \ \mathcal{P}(y) \ \ \mathcal{P}(\iota y Q) \ \ \mathcal{P}(x,y) \ \ \mathcal{P}(y,y)$$

Following are the definitions of '*term*', '*statement*' and '*formula*'. Due to their use in *ATIS*, all variables are *bound*.

(1) *term* $=_{df}$

    (i) $x_1 \ x_2 \ \dots \ x_n \ x \ y$; where "…" has its accepted meaning

    (ii) $\iota x \mathcal{P}$

    (iii) $\hat{w} \mathcal{P}$

(2) *statement* $=_{df}$

    (i) $A \in B$, where 'A' and 'B' are terms, 'A' is a *component* and 'B' is a *class*, since only sentences concerned exclusively with classes are considered to be statements

    (ii) $\forall x \mathcal{P}$, where 'x' is a variable and '$\mathcal{P}$' is a statement

    (iii) $\sim \mathcal{P}$, where '$\mathcal{P}$' is a statement

    (iv) $\mathcal{P} \wedge \mathcal{Q}$, where '$\mathcal{P}$' and '$\mathcal{Q}$' are statements

(3) *formula* $=_{df}$

    (i) S, where 'S' is comprised of a sequence of statements constructed with '$\sim$' and '$\wedge$'

(4) $\alpha = \beta \ \ =_{df} \ \forall x (x \in \alpha \equiv x \in \beta) \ =_{df} \ \alpha = \beta \ =_{df} \ \alpha =_x \beta$

'$\mathcal{P}$' is referred to as the scope of the quantifiers.

Definition (4) defines equality of sets. The last notation, $\alpha =_x \beta$, is very useful in *ATIS*. Due to the complexity of systems, it may be that various properties are defined with respect to the same set of components. Rather than having to consider numerous sets, the properties can be defined with respect to a specific subset. For example, one may wish to determine the behavior of a system with respect to various subsets. Such can be designated as follows: $\mathfrak{S} =_B \mathcal{V}$; $\mathfrak{S} =_C \mathcal{L}$; and $\mathfrak{S} =_D \mathcal{G}$.

Then an APT analysis can be performed on the following set: $\ell = \{\mathcal{V}, \mathcal{L}, \mathcal{G}\}$.

With the foregoing definitions, we now have the following axiom schemas extended from the *Predicate Calculus* to include the axiom schema for the *Class Calculus*, Axiom (12).

Transformation Rule, Modus Ponens: $\mathcal{P}, \mathcal{P} \supset \mathcal{Q} \vdash \mathcal{Q}$

(1) $\mathcal{P} \supset \mathcal{P}\,\mathcal{P}$

(2) $\mathcal{P}\,\mathcal{Q} \supset \mathcal{P}$

(3) $(\mathcal{P} \supset \mathcal{Q}) \vee [\sim(\mathcal{Q}\mathcal{R}) \supset \sim(\mathcal{R}\mathcal{P})]$

(4) $\forall x(\mathcal{P} \supset \mathcal{Q}) \supset (\forall x \mathcal{P} \supset \forall x \mathcal{Q})$

(5) $\mathcal{P} \supset \forall x \mathcal{P}$

(6) $\forall x \mathcal{P}(x,y) \supset \mathcal{P}(y,y)$

       i. $\forall x \mathcal{P}(x) \supset \mathcal{P}(y)$

(7) $\forall x,y,z[(x = y) \supset (x \in z \supset y \in z)]$

(8) $\forall x_1,x_2,\ldots,x_n(\forall x \mathcal{P}(x) \supset \mathcal{P}(\iota y \mathcal{Q})$

(9) $\forall x_1,x_2,\ldots,x_n[\forall x(\mathcal{P} \equiv \mathcal{Q}) \supset (\iota x \mathcal{P} = \iota x \mathcal{Q})]$

(10) $\forall x_1,x_2,\ldots,x_n[\iota x \mathcal{P}(x) \equiv \iota y \mathcal{P}(y)]$

       i. $\forall x_1,x_2,\ldots,x_n[\iota x \mathcal{P} = \iota y \mathcal{Q}]$

(11) $\forall x_1,x_2,\ldots,x_n[\exists^1 x \mathcal{P} \supset (\forall x[\iota x \mathcal{P} = x \equiv \mathcal{P}])]$

       i. $\forall x_1,x_2,\ldots,x_n[\exists^1 x \mathcal{P}(x) \supset (\forall x[\iota x \mathcal{P}(x) = x \equiv \mathcal{P}(x)])]$

      ii. $\forall x_1,x_2,\ldots,x_n[\exists^1 x \mathcal{P}(x) \supset (\forall y[\iota x \mathcal{P}(x) = y \equiv \mathcal{P}(y)])]$

(12) $\exists y \forall x(x \in y \equiv \mathcal{P})$

Axiom Schemas (1) to (3) are the valid-value axioms of the *Sentential Calculus*.

Axiom Schemas (4) to (6) allow for generalization from $\vdash \mathcal{P}$ to $\vdash \forall x \mathcal{P}$.

Axiom Schema (6) provides for substitution of a value for a variable.

Axiom Schema (7) allows for substitution of equivalent terms resulting from equality.

Axiom Schemas (8) to (11) are the axiom schemas for ι.

Axiom Schema (8) asserts that if $\mathcal{P}(x)$ is true for all x, then $ιy\mathcal{Q}$ is the name of one of those objects.

Axiom Schema (9) asserts that if $\mathcal{P}$ and $\mathcal{Q}$ are equivalent for all x, then $ιx\mathcal{P}$ and $ιx\mathcal{Q}$ are names of the same object.

Axiom Schema (10) allows for change of variables.

Axiom Schema (11) asserts that if there is a unique x that makes $\mathcal{P}$ valid, then $ιx\mathcal{P}$ is that x.

Axiom Schema (12) is the schema that introduces classes. This axiom allows for the *Set Calculus* to be integrated into the formal theory.

Relation Calculus

For *ATIS*, we are concerned with attempting to use as many mathematical constructs as possible while clearly describing the desired system properties.

While mathematics is frequently concerned with *functions*, for *ATIS* the concerns may be directed more toward *relations*.

However, while functions are normally considered as being single-valued, many-valued functions are possible. The relation $[\![x,x^2]\!]$ is a multi-valued function. $[\![x,x<y]\!]$ also is a multi-valued function. These are well-defined functions since the ordered pairs that define the functions are well-defined. Whether or not these are considered functions or relations is not clear; that is, there does not seem to be any clear distinction between the two. With '*function*' being restricted to *single-valued functions*, these examples would be considered as *relations*. One distinction has been that '*function*' was restricted to relations that resulted in well-defined curves, whereas '*relation*' would be for those statements that defined all other characterizations. Thus, $[\![x,x^2]\!]$ would be a *function*, and $[\![x,x<y]\!]$ would be a *relation*.

In *ATIS,* the distinction between '*function*' and '*relation*' will not be considered. The only question is whether or not the appropriate mathematical construct clearly portrays the system characteristic being considered. It appears as though most of the concerns for *ATIS* will be with respect to *morphisms*; that is, *relational mappings*. Whether such mappings are '*functions*' or '*relations*' is moot. If a single-valued function is required, then such can be stated. For purposes of analysis, morphisms or relations will be considered, since functions are a special type of relation. Further, where the "function notation" is used, it is not to be construed as restrictive. Normally, it will probably designate a single-valued function, but such in this theory is not required. Either the context or by definition, the type of function will be determined.

The *Relation Calculus* for *ATIS* is concerned with the affect relations that define a system and the morphisms that characterize the properties of the system as derived from those affect relations.

The *Relation Calculus* axiom schemas will be presented first. This will complete the presentation of the formal logic.

Following the presentation of the formal logic, the content required for a *General System Theory* will be introduced. First, the axiom that asserts the existence of a *General System* will be introduced. Then the axioms that establish the empirical systems that are to be analyzed and the criteria for such analysis will be given.

We have already introduced the notation that will be used to identify a class or set of objects, or components, $\hat{w}\mathcal{P}(w)$. Now the characterization of those components will be discussed.

Whereas x identifies a single component within the set, it may be that we wish to identify an object that consists of two or more components. The following notations will be used to identify such sets.

'$\{x, y\}$' identifies a component of a set that consists of two single components.

If it is desired to specify that the set consists only of binary-components, then the following notation will so indicate: $\hat{w}^2\mathcal{P}(w)$. This notation designates that the class or set of components consists only of sets each of which contains two single components.

Hence, '$\hat{w}^2\mathcal{P}(w)$' designates a *family* of binary sets.

By extension, '$\hat{w}^n\mathcal{P}(w)$' designates a family of sets, each member of which contains n components; that is, $\{x_1, x_2, …, x_n\}_i \in w$.

For affect relations, an additional type of set will be required. This set will contain binary-components and a set that contains one of the binary-components in a unary-component set. That is, the set will be configured by the following representation:

$$\{\{x\}, \{x, y\}\}$$

Where each unary-, binary-component set is included and no other sets are included. This notation is frequently represented by the ordered pair: (x,y).

For this set, the class quantifier will be represented as: $\hat{w}^{2|1}\mathcal{P}(w)$.

By extension, '$\hat{w}^{n|n-1|…|1}\mathcal{P}(w)$' designates a family of sets that include all and only those ordered subsets of the largest set. For n = 4, the family of sets would be characterized by components of the form: $\{\{a\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\} = (a,b,c,d)$, an **ordered** 4-tuple.

With the foregoing definitions, we now have the following axiom schema for the *Relation Calculus*, Axiom (13), which introduces relations.

Axiom (13) $\exists z \forall x,y(\{x,y\} \in z \land \{x\} \in z \equiv \mathcal{R}$

The following axiom schemas provide for substitution within and identification of relations.

Axiom (14) $\forall x \mathcal{R}(x) \supset \mathcal{R}(\iota y \mathcal{Q})$

Axiom (15) $\forall x (\mathcal{R} \equiv \mathcal{Q}) \supset (\iota x \mathcal{R} \equiv \iota x \mathcal{Q})$

Axiom (16) $\iota(x,y)\mathcal{R}(x,y) \equiv \iota(p,q)\mathcal{R}(p,q))$

Axiom Schema (14) asserts that if $\mathcal{R}(x)$ is true for all x, then $\iota y \mathcal{Q}$ is the name of one of those relations.

Axiom Schema (15) asserts that if relations $\mathcal{R}$ and $\mathcal{Q}$ are equivalent for all x, then $\iota x \mathcal{R}$ and $ix \mathcal{Q}$ are names of the same relation. This is a critical axiom for determining morphisms.

Axiom Schema (16) allows for change of variables.

ATIS **Calculus**

In the preceding sections, the formal logic has been established.

Now, the calculus must be developed that begins to provide the substance for the desired theories. These theories are descriptive of what will be called *General Systems*. Therefore, the first axiom will introduce the characteristics of a *General System*.

The development of the calculus that results in the empirical theory is dependent upon the concept of an *Options Set*. The *Options Set* is that listing of *Properties* and *Associated Axioms* that will result in a system-descriptive theory that will be analyzed pursuant to the derived formal logic herein established. The specific *Options Set* herein developed is the *ATIS Options Set*. This set will consist of the derived list of properties, and all *General System* axioms that are associated with those properties.

An analysis of a system is obtained by determining those properties that are descriptive of the system. Those properties are then identified in the *ATIS Options Set*. Following identification of these properties, the *Associated Axioms* are then selected. *Associated Axioms* are those in which one or more of the selected properties occur. With the selection of these axioms, an analysis of the system is possible using the *Predicate* and *Relation Calculi* herein developed.

It is also intended that a topological analysis will eventually be possible either by the direct use of operations taken from mathematical topology or a derivation thereof. Such an analysis, along with other analytic techniques, is beyond the scope of this report.

The following axiom asserts that if we have a set of a specific system defined by components and a set of that system defined by relations of those components, then we have a *General System* that can be characterized by *ATIS*. Axiom Schema (17) is the *General System* axiom scheme.

Axiom (17) $\hat{w}\mathcal{P}(w) \equiv \mathbb{S}_{\textbf{x}} \wedge \hat{y}\mathcal{P}(y) \equiv \mathbb{S}_{\phi} \equiv \mathcal{G}(\mathbb{S}_{\textbf{x}}, \mathbb{S}_{\phi})$

The affect relations, $\mathbb{S}_{\phi}$, determine the properties, $\mathcal{P}$, of an *Intentional General System*.

Axiom (18) asserts that if we have an *Intentional General System*, $\mathscr{A}\mathcal{G}$, then for every property, $\mathcal{P}$, there exists a property qualifier that determines the class $\hat{w}\mathcal{P}(w)$.

Axiom (18) $\mathscr{A}\mathcal{G} \supset \forall \mathcal{P}(w) \exists \hat{w}(\hat{w}\mathcal{P}(w))$

The following axiom asserts that if we have a property class then there is a morphism that can be defined between that class and another property class.

Axiom (19) $\hat{w}\mathcal{P}(w) \supset \exists \breve{\mathcal{X}} \exists \hat{y}\mathcal{P}(y)(\breve{\mathcal{X}}(\hat{w}\mathcal{P}(w) \rightarrow \hat{y}\mathcal{P}(y)))$

ATIS **Options Set Defined**

The *ATIS Options Set*, $\mathcal{A}$, is defined by the set of system properties, $\mathcal{P}$, and system Affect Relations, $\mathcal{A}$.

$$\mathcal{A} =_{df} \hat{w}_{i=1:n} \mathcal{P}_i(w_i) \cup \hat{y}_{j=1:n} \mathcal{A}_i(y_j)$$

Definitions of Logical Operations in Proofs

In addition to the logical schemas presented above, some proofs of theorems may require an application of the definition of the logical operations. An example is given below.

The following operations were previously defined:

Definition. $\mathcal{P} \vee \mathcal{Q} =_{df} \sim(\sim\mathcal{P} \wedge \sim\mathcal{Q})$

Definition. $\mathcal{P} \supset \mathcal{Q} =_{df} \sim(\mathcal{P} \wedge \sim\mathcal{Q})$

Definition. $\mathcal{P} \equiv \mathcal{Q} =_{df} (\mathcal{P} \supset \mathcal{Q}) \wedge (\mathcal{Q} \supset \mathcal{P}) \; .\equiv. \; \sim(\mathcal{P} \wedge \sim\mathcal{Q}) \wedge \sim(\mathcal{Q} \wedge \sim\mathcal{P})$

The following theorem demonstrates an application of the use of definitions in the proof of a theorem.

Theorem. $\vdash_{HO} \mathcal{C}^{c\uparrow} \vee_{F} \mathcal{C}^{c\downarrow} \supset_{S} \mathcal{C}^{c\downarrow}$

"If hierarchical order is constant or increasing, or if flexibility is constant or decreasing, then strongness is constant or decreasing."

Proof:

1. $_{S}\mathcal{C}^{\uparrow} \supset_{HO} \mathcal{C}^{\downarrow}$ — Axiom 55; i.e., "If strongness increases, then hierarchical order decreases."

2. $_{S}\mathcal{C}^{\uparrow} \supset_{F} \mathcal{C}^{\uparrow}$ — Axiom 56; i.e., "If strongness increases, then flexibility increases."

3. $_{S}\mathcal{C}^{\uparrow} \supset_{HO} \mathcal{C}^{\downarrow} \wedge_{F} \mathcal{C}^{\uparrow}$ — Logical Schema 4

4. $\sim(_{HO}\mathcal{C}^{\downarrow} \wedge_{F} \mathcal{C}^{\uparrow}) \supset \sim_{S}\mathcal{C}^{\uparrow}$ — Logical Schema 5

5. $_{HO}\mathcal{C}^{c\uparrow} \vee_{F} \mathcal{C}^{c\downarrow} \supset \sim_{S}\mathcal{C}^{\uparrow}$ — Definition of '∨'

6. $_{HO}\mathcal{C}^{c\uparrow} \vee_{F} \mathcal{C}^{c\downarrow} \supset_{S} \mathcal{C}^{c\downarrow}$ — Logical Equivalence of '~'

7. $\vdash_{HO} \mathcal{C}^{c\uparrow} \vee_{F} \mathcal{C}^{c\downarrow} \supset_{S} \mathcal{C}^{c\downarrow}$ — Q.E.D.[32]

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[32] "Q.E.D." comes from the Latin *quod erat demonstrandum*, "that which was to be demonstrated"; or, in mathematics, "that which was to be proved."

Axiom 181 is a Theorem

With the number of axioms presented for the theory, it is possible that some of the axioms are in fact theorems; that is, they are derivable from the other axioms. Such is the case with Axiom 181.

Axiom 181 states: $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+c} \supset {}_{C}\mathcal{C}^{\downarrow}$.

That is: "If size increases and complexity growth is constant, then centrality decreases."

This statement will now be proved as a theorem.

Theorem 181. $\vdash \mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+c} \supset {}_{C}\mathcal{C}^{\downarrow}$

Proof:

1.	$\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+c} \supset T_{\mathcal{P}}^{\uparrow}$	Axiom 194; i.e., "If size increases and complexity growth is constant, then toput increases."
2.	$T_{\mathcal{P}}^{\uparrow} \supset {}_{C}\mathcal{C}^{\downarrow}$	Axiom 90; i.e., "If toput increases, then centrality decreases."
3.	$\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+c} \supset {}_{C}\mathcal{C}^{\downarrow}$	Logical Schema 0 (Transitive Property) on Steps 1 and 2
4.	$\vdash \mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+c} \supset {}_{C}\mathcal{C}^{\downarrow}$	Q.E.D.

Inconsistent Axioms

Certain axioms of the SIGGS Theory have or will be found to be inconsistent. That is, they are inconsistent when combined within the same theory. This does not mean that either axiom is "wrong" or "false" or "not-valid." It simply means that they cannot be taken together in the same theory. No determination will be made at this time as to which axiom is more appropriate for the theory. It may simply be that several theories will be developed from the SIGGS Theory axioms.

The following pairs of axioms have been found to be inconsistent and will be so proved below: Axioms 55 and 112, Axioms 90 and 106, and Axioms 175 and 183.

Theorem 55-112. $\vdash \sim({_S}\mathscr{C}^\uparrow \supset {_{HO}}\mathscr{C}^\downarrow :\equiv: {_S}\mathscr{C}^\uparrow \wedge {_{HO}}\mathscr{C}^c \supset {_R}\mathscr{R}^\downarrow)$

That is: "$\vdash \sim$(Axiom 55 $:\equiv:$ Axiom 112)"

Proof:

1.	$_S\mathscr{C}^\uparrow \supset {_{HO}}\mathscr{C}^\downarrow$	Axiom 55; i.e., "If strongness increases, then hierarchical order decreases."
2.	$_S\mathscr{C}^\uparrow \vdash {_{HO}}\mathscr{C}^\downarrow$	Deduction Theorem on 1
3.	$_S\mathscr{C}^\uparrow$	Assumption from 2
4.	$_{HO}\mathscr{C}^\downarrow$	Modus Ponens on 3 and 1
5.	$_S\mathscr{C}^\uparrow \wedge {_{HO}}\mathscr{C}^c \supset {_R}\mathscr{R}^\downarrow$	Axiom 112; i.e., "If strongness increases and hierarchical order is constant, then regulation decreases."
6.	$_S\mathscr{C}^\uparrow \supset. {_{HO}}\mathscr{C}^c \supset {_R}\mathscr{R}^\downarrow$	Logical Equivalence 3 on 5
7.	$_S\mathscr{C}^\uparrow \vdash {_{HO}}\mathscr{C}^c \supset {_R}\mathscr{R}^\downarrow$	Deduction Theorem on 6
8.	$_S\mathscr{C}^\uparrow, {_{HO}}\mathscr{C}^c \vdash {_R}\mathscr{R}^\downarrow$	Deduction Theorem on 7
9.	$_{HO}\mathscr{C}^c$	Assumption from 8
10.	$_{HO}\mathscr{C}^\downarrow \wedge {_{HO}}\mathscr{C}^c$	From Steps 4 and 9
11.	$_{HO}\mathscr{C}^\downarrow \wedge {_{HO}}\mathscr{C}^c$	Contradiction of '$\wedge$'
12.	$\vdash \sim({_S}\mathscr{C}^\uparrow \supset {_{HO}}\mathscr{C}^\downarrow :\equiv: {_S}\mathscr{C}^\uparrow \wedge {_{HO}}\mathscr{C}^c \supset {_R}\mathscr{R}^\downarrow)$ Q.E.D.	

Theorem 90-106. $\vdash \sim(T_{\wp}^{\uparrow} \supset\ _{C}\mathscr{C}^{\downarrow} :\equiv:\ _{C}\mathscr{C}^{\uparrow} \vee\ _{S}\mathscr{C}^{\uparrow} \supset T_{\wp}^{\uparrow})$

That is: "$\vdash \sim$(Axiom 90 $:\equiv:$ Axiom 106)"

Proof:

1.	$T_{\wp}^{\uparrow} \supset\ _{C}\mathscr{C}^{\downarrow}$	Axiom 90; i.e., "If toput increases, then centrality decreases."
2.	$T_{\wp}^{\uparrow} \vdash\ _{C}\mathscr{C}^{\downarrow}$	Deduction Theorem on 1
3.	$T_{\wp}^{\uparrow}$	Assumption from 2
4.	$_{C}\mathscr{C}^{\downarrow}$	Modus Ponens on 3 and 1
5.	$_{C}\mathscr{C}^{\uparrow} \vee\ _{S}\mathscr{C}^{\uparrow} \supset T_{\wp}^{\uparrow}$	Axiom 106; i.e., "If complete connectivity increases or strongness increases, then toput increases."
6.	$_{C}\mathscr{C}^{\uparrow} \supset T_{\wp}^{\uparrow}$	Assumption of Case on 5
7.	$_{C}\mathscr{C}^{\uparrow} \vdash T_{\wp}^{\uparrow}$	Deduction Theorem on 6
8.	$_{C}\mathscr{C}^{\uparrow}$	Assumption from 7
9.	$_{C}\mathscr{C}^{\downarrow} \wedge\ _{C}\mathscr{C}^{\uparrow}$	From Steps 4 and 8
10.	$_{C}\mathscr{C}^{\downarrow} \wedge\ _{C}\mathscr{C}^{\uparrow}$	Contradiction of '$\wedge$'
11.	$\vdash \sim(T_{\wp}^{\uparrow} \supset\ _{C}\mathscr{C}^{\downarrow} :\equiv:\ _{C}\mathscr{C}^{\uparrow} \vee\ _{S}\mathscr{C}^{\uparrow} \supset T_{\wp}^{\uparrow})$ Q.E.D.	

Theorem 175-183. $\vdash \sim(X^{-\uparrow} \supset Z^{-\uparrow} \vee {}_D\mathscr{C}^{\uparrow} :\equiv: Z^{\downarrow} \wedge X^{-\uparrow} \supset {}_D\mathscr{C}^{\downarrow})$

That is: "$\vdash \sim$(Axiom 175 $:\equiv$: Axiom 183)"

Proof:

1.	$X^{-\uparrow} \supset Z^{-\uparrow} \vee {}_D\mathscr{C}^{\uparrow}$	Axiom 175; i.e., "If complexity degeneration increases, then size degeneration increases or disconnectivity increases."
2.	$X^{-\uparrow} \vdash Z^{-\uparrow} \vee {}_D\mathscr{C}^{\uparrow}$	Deduction Theorem on 1
3.	$X^{-\uparrow}$	Assumption from 2
4.	$Z^{-\uparrow} \vee {}_D\mathscr{C}^{\uparrow}$	Modus Ponens on 3 and 1
5.	${}_D\mathscr{C}^{\uparrow}$	Assumption of Case on 4
6.	$Z^{\downarrow} \wedge X^{-\uparrow} \supset {}_D\mathscr{C}^{\downarrow}$	Axiom 183; i.e., "If size decreases and complexity degeneration increases, then disconnectivity decreases."
7.	$Z^{\downarrow} \wedge X^{-\uparrow} \vdash {}_D\mathscr{C}^{\downarrow}$	Deduction Theorem on 4
8.	$Z^{\downarrow} \wedge X^{-\uparrow}$	Assumption from 5
9.	${}_D\mathscr{C}^{\downarrow}$	Modus Ponens on 6 and 4
10.	${}_D\mathscr{C}^{\uparrow} \wedge {}_D\mathscr{C}^{\downarrow}$	Steps 5 and 9
11.	${}_D\mathscr{C}^{\uparrow} \wedge {}_D\mathscr{C}^{\downarrow}$	Contradiction of '$\wedge$'
12.	$\vdash \sim(X^{-\uparrow} \supset Z^{-\uparrow} \vee {}_D\mathscr{C}^{\uparrow} :\equiv: Z^{\downarrow} \wedge X^{-\uparrow} \supset {}_D\mathscr{C}^{\downarrow})$	Q.E.D.

Hypothesis-Based vs. Axiom-Based Research Methodologies

The Hypothesis and Axiom Distinction

Social scientists rely on hypothesis-based research and attempt to use that methodology to develop social theories. To understand the fallacy of such an approach, the distinction between *hypothesis* and *axiom* must be understood.

In the social sciences, *axioms* and *hypotheses* are frequently considered to have the same meaning. However, in theory development, these two terms are distinctly different.

In fact, the distinctions between *axiom* and *hypothesis* provide strong confirmation why there has been no comprehensive theory developed for the social sciences, and why hypothesis-driven research <u>cannot</u> provide a basis for any such theory development.

Essentially, the distinction is that a *hypothesis* is a conjecture about an observation or a perceived empirical event that is stated as a conclusion of fact that is to be *validated* by experimental testing. An *axiom*, on the other hand, is a statement that relates properties of a theory, or the components (objects) of a theory to its properties. An *axiom* is theory-based; a *hypothesis* is empirical-based.

When a hypothesis is stated, there is no intent that it is meant to develop theory—it is meant to be validated as an assertion of fact. Social scientists then attempt, after-the-fact to utilize the results of the testing of the hypothesis to somehow develop theory. However, since the validation of the hypothesis is nothing more than that, there is nothing by which the hypothesis can be related to other properties or hypotheses that could result in theory development.

Very simply, hypotheses are not designed to develop theory.

The Writing of a Hypothesis as Opposed to an Axiom

Consider the following statement:

HYPOTHESIS: Student choice and independence are the primary motivators for learning.

As stated, this is a hypothesis. It is stated as a conclusion of fact that is to be validated. If it is validated, it provides no relevant relation to any other statements that might be part of a theory and there are no leading assertions from which additional theory statements can be derived. This is so even if the statement is framed as an implication as follows:

HYPOTHESIS: If student choice and independence are related to learning, then establishing student choice and independence in the classroom will confirm them as the primary motivators for learning.

Now consider the following statement:

AXIOM: Students are independent systems (where *independent system* is a property defined by a systems theory).

This statement is an axiom as it relates the components of a theory, *students*, to a property of the theory, *independent system*. It is not a conclusion of fact, as there is nothing to validate, but a theoretical construct, an axiom for a theory, that informs us about a theory property by which students are identified— that is, it is <u>assumed</u> that *students* are *independent systems*. Further, this statement is neither "true" nor "false"—it is simply an assertion that is <u>assumed</u> to be *valid*. Further, there is no amount of testing that can ever confirm the validity of this assertion since there is nothing to validate. Whether or not this axiom is *valid* is not the issue, since it is *assumed* that it is. The issue is whether or not the theorems (or *logically-derived, theory-based hypotheses*) that are deductively obtained from it and other axioms of the theory are validated. Validation of theorems (*logically-derived, theory-based hypotheses*) derived from this axiom provides the on-going "preponderance of evidence" that the axiom is a warranted valid assertion within the theory.

An additional point needs to be made about this axiom. It may be contended, as is often done in the social sciences, that if it is found that there is a student who is not an "independent system," then the axiom is demonstrated to be "false" and the entire theory must be discarded. To the contrary, such an approach is applicable to hypotheses but not to axioms. If an empirical example is found that refutes a hypothesis, then the research is complete and the hypothesis is rejected. However, an axiom is a basic assumption, not a hypothesis. If a student is found not to be an "independent system," as *independence* is defined within the theory, then that student simply is not considered with respect to the theory—the criteria for the basic assumption has not been met. A comparable example from geometry would be where an artist considers parallel lines to meet at "infinity" whereas in Euclidean geometry they do not. Very simply, the one geometry does not refute the other; they are simply two different geometries. If a student is not an "independent system," then that student is not part of the class of students that are being analyzed with respect to the theory that contains the above axiom. When it is asserted that there is nothing to validate with respect to the axiom that is exactly what is meant—the axiom is assumed to be valid and all analyses proceed upon that assumption. If the empirical evidence demonstrates that a particular event does not meet the criteria for the axiom, then all that means is that that event is not analyzed with respect to the theory being considered. This should be a welcome outcome for all educologists who believe that students should be treated "individually"—they in fact are members of distinct systems that require distinct theories! [Such distinct theories are provided for by the *Options Set* of *ATIS* (*Axiomatic Theory of Intentional Systems*).]

Now, if as a result of the definition of *independent system* along with other system properties and axioms it is determined that "individual choice and independence are motivators for learning," then it is as a result of some theory derivation, and not an *a priori* assertion of fact. Then, through various empirical analyses it can be determined whether or not in fact "choice and independence" are the "*primary* motivators for learning." However, even the validation of this conclusion, should it actually be derived, will depend on all of the assumptions and qualifications it took to arrive at this conclusion. Tests are not set up at the discretion and "creativity" of a researcher, but are determined by the parameters of the theory.

When designing tests for hypotheses, the burden is on the researcher to determine if all parameters have been accounted for; hence, the frequent occurrences where two evaluations of the "same" hypothesis results in different outcomes. With an axiomatic theory that results in the derivation of a theorem, it is the axioms and theorems that dictate the parameters of the experiment. Have all assumptions of the theory been accounted for in the design of the experiment?

The distinction between *axiom* and *hypothesis* is seen to be quite profound for theory development, and it is important to keep clear their differences.

Examples of Hypotheses in the Social Sciences and Converting Them to Axioms

Cognitive Load Theory

To see the distinction between hypotheses and axioms and how the latter may lead to theory, consider the following hypothesis taken from the social sciences as propounded by Tracey Clarke, Paul Ayres, and John Sweller in "The Impact of Sequencing and Prior Knowledge on Learning Mathematics through Spreadsheet Applications" (Clarke, 2005):

HYPOTHESIS: Students with a low-level knowledge of spreadsheets learn mathematics more effectively if the relevant spreadsheet skills are learned prior to attempting the mathematical tasks.

The results of testing supported this hypothesis. For our purposes the greater concern is how this hypothesis was developed and whether it may lead to theory development. The rationale for the hypothesis is stated by the researchers as follows:

> According to cognitive load theory, instruction needs to be designed in a manner that facilitates the acquisition of knowledge in long-term memory while reducing unnecessary demands on working memory. When technology is used to deliver instruction, the sequence in which students learn to use the technology and learn the relevant subject matter may have cognitive load implications, and should interact with their prior knowledge levels. An experiment, using spreadsheets to assist student learning of mathematics, indicated that for students with little knowledge of spreadsheets, sequential instruction on spreadsheets followed by mathematics instruction was superior to a concurrent presentation. These results are explained in terms of cognitive load theory. (p. 15)

The process by which this hypothesis was developed is a retroductive process; that is, cognitive load theory was used as a model to develop assertions about learning mathematics. However, there is confusion concerning this process since it is claimed: "These results are explained in terms of cognitive load theory" (see last sentence from above quotation).

If in fact the results "are explained in terms of cognitive load theory," and *Cognitive Load Theory* (CLT) is in fact a theory, then this hypothesis is a theorem of CLT and should be deductively obtained from that theory as a theorem to be validated; or, possibly, it is an interpretation of CLT and was derived as an abductive process—that is CLT was used as a model of mathematics learning and the content of the desired hypothesis was substituted for the content of CLT.

However, it is not claimed that either approach was used, so the question moves to whether or not CLT is actually a theory, or is it a hypothesis that has been validated through various tests? If it is a theory, then we may be able to determine in what sense it is claimed that CLT "explains" the hypothesis.

First, the use of the term *theory* in the context of CLT indicates that it is not either a formal theory or an axiomatic theory. If it is a theory, then it appears to be a descriptive theory. But, even as a descriptive theory, it appears to be very limited in scope and functions more like a hypothesis since deductive derivations are difficult to obtain. But, let us look at this more carefully.

Cognitive Load Theory, developed by J. Sweller, is founded on the following four principles:

- Working memory, or short-term memory, has a maximum capacity identified as *maximum cognitive load*
- Information that exceeds *maximum cognitive load* is lost
- Learning requires that *cognitive load* remain below some value that is less than *maximum cognitive load*
- Long-term memory is consciously processed through working memory

NOTE: The "theory" and "axioms" presented below are for the sole purpose of demonstrating a possible construction of an axiomatic theory and in no way is to be construed as a replacement for the *Cognitive Load Theory* developed by J. Sweller. In fact, it is only as a result of the careful development of CLT that it is possible to derive an axiomatic theory therefrom. In general, it is very difficult to ascertain axioms from a descriptive theory, since they frequently are so vaguely worded that explicit statements of their assumptions are difficult or impossible to determine. Fortunately, CLT is not such a theory.

Provided below is a preliminary development for a *Theory of Memory and Learning* that is retroductively-derived from CLT. Briefly presented are the primitive terms, initial axioms, definitions and a few theorems of the theory.

Theory of Memory and Learning

PRIMITIVE TERMS: *Cognition*, *memory*, *working memory*, *consciously*, *cognitive load*, *mental structures*, *patterns*, and *languages*

> **AXIOM 1:** Working memory is that memory which is used to consciously process information.
>
> **AXIOM 2:** Working memory has a maximum capacity identified as *maximum cognitive load*.
>
> **AXIOM 3:**   Working memory that is maintained below *maximum cognitive load* results in short-term memory acquisition.
>
> **AXIOM 4:** Long-term memory is consciously processed through working memory.
>
> **AXIOM 5:** Cognition is determined by a sequence of recognizable patterns or languages.
>
> **AXIOM 6:** Long-term memory is short-term memory that is processed and related to an existing or newly developed cognitive schema, or structure.

DEFINITION 1: 'Cognitive schemas' are memory constructs that map short-term memory cognition onto devised mental structures that interpret immediate cognition.

DEFINITION 2: 'Learning' is defined as that processed cognitive load that results in the acquisition of short-term memory.

As a result of these axioms and definitions, we obtain the following theorems:

THEOREM 1: Information that exceeds *maximum cognitive load* is not cognizable.

> **PROOF OF THEOREM 1:**
> - Working memory has a maximum capacity identified as *maximum cognitive load*. (Axiom 2.)
> - That which exceeds maximum capacity is not cognizable. (Definition of 'maximum'.)

Another way of stating Theorem 1 is:

THEOREM 1: Information that exceeds *maximum cognitive load* is lost.

This statement of the theorem is the second statement of the four principles cited above for CLT.

THEOREM 2: For learning to occur, *cognitive load* must remain below *maximum cognitive load*.

> **PROOF OF THEOREM 2:**
> - Cognitive load that exceeds maximum cognitive load is not cognizable and, therefore, not processed. (Theorem 1.)
> - Working memory that is maintained below *maximum cognitive load* results in short-term memory acquisition. (Axiom 2.)
> - Short-term memory acquisition results in learning. (Definition of 'learning'.)

Now the problem is to determine if the hypothesis relating to learning mathematics considered previously can be derived from this theory.

Stating the hypothesis again:

HYPOTHESIS / THEOREM 3: Students with a low-level knowledge of spreadsheets learn mathematics more effectively if the relevant spreadsheet skills are learned prior to attempting the mathematical tasks.

> **PROOF OF THEOREM 3:**
> - Students do not have cognitive schemas relating to spreadsheets. (Assumption of Theorem 3.)
> - Students do not have cognitive schemas relating to mathematics. (Assumption of Theorem 3.)
> - Lack of cognitive schemas precludes long-term memory. (Axiom 6.)
> - Spreadsheet cognition precedes mathematics cognition. (By Axiom 5 and assumption of Theorem 3, the spreadsheet structure provides the basic "language" by which mathematics is learned.)
> - A spreadsheet cognitive schema must be developed for long-term memory to take place. (Axiom 6.)
> - Therefore, relevant spreadsheet skills must be learned prior to the learning of mathematical tasks. (Conclusion of Theorem 3.)

The importance of this axiomatic development is that now a much stronger claim can be made concerning Theorem 3. Whereas the initial researchers could only claim: "These results are explained in terms of cognitive load theory," it can now be claimed more strongly: "These results are deductively obtained from the *Theory of Memory and Learning* and are validated by empirical testing."

But what is the far-reaching effect of this second approach? By validating Theorem 3 the researchers have not only validated their "hypothesis" (theorem) but have now provided support for the theory. This validation has now initiated a process that, hopefully, will eventually provide a "preponderance of evidence" that the theory consistently provides valid outcomes. By framing CLT as an axiomatic theory, every validation of a theorem (or *logically-derived, theory-based hypothesis* if you want) validates not only the theorem but the theory. Eventually, we will be able to obtain theorems deductively from the theory and proceed with confidence that the outcome is accurate, with or without further validation. This is very important, since otherwise every hypothesis must be continually validated in every new setting, in every new school, in every new learning environment.

Whereas the hypothesis has been validated for this one group of students learning mathematics from a spreadsheet, what can we say if instead of a spreadsheet, new computer software is utilized? Will they have to learn the software before learning the mathematics? At first glance, the answer should be "obvious" even without any testing. But, for the sake of making a point, the point is also "obvious"—we have already provided the proof that they would have to learn the software and we do not have to, once again, conduct testing to validate the theorem.

But now, what about results that are not so obvious?

Theory of Memory and Learning—Applications

Theorem 3 provided content that is not stated in the theory axioms. Applications of this theory are the result of the logical process of abduction; that is, the theory content is determined independent of the theory and substituted for the theoretical constructs. For example, "working memory" of the theory is replaced by "spreadsheet" and "mathematics" by the specific application. Additional theory applications can be obtained by interpreting various cognitive schemas.

For a non-obvious theory outcome, consider the following schemas that have been established as part of long-term memory:

(1) Learned behavior described as the schema "assertive"; and
(2) Learned behavior described as the schema "attention-to-detail."

When students learn to keyboard it is frequently asserted that in order to improve speed and accuracy they must practice keyboarding. However, if that were accurate, then anyone who has been keyboarding for many years should be doing so at approximately 60 words-per-minute with great accuracy—whereas this is not the case. There must be more to developing speed and accuracy than practicing keyboarding. Here it is noted that empirical observation refuted the prevailing hypothesis. The alternative was then not derived from the empirical observation, but from recognition that the *Theory of Memory and Learning* should apply to this empirical event—a retroductive process. From the *Theory of Memory and Learning* it is determined that these students have developed certain cognitive schemas defined as *assertive* and *attention-to-detail* that are independent of content.

As a result of these cognitive schemas and the process of abduction by which theory content is determined, the following theorem is obtained:

THEOREM 4: Keyboarding speed can be improved by any off-task activity that increases one's *assertiveness*; and keyboarding accuracy can be improved by any off-task activity that increases one's *attention-to-detail*.

Theorem 4 is a direct result of Axiom 6 and Axiom 5. Theorem 4 is a non-obvious result of the *Theory of Memory and Learning* that was derived from *Cognitive Load Theory*. While there is anecdotal evidence that Theorem 4 is valid, actual validation or refutation of Theorem 4 is left to those who are more skilled at constructing appropriate tests. Whether Theorem 4 is found to be valid or not, the efficacy of an axiomatic theory has been demonstrated as being one that results in non-obvious conclusions. And, as seen here, <u>an axiomatic theory does **not** have to be formal</u>, although the formalization of this theory may result in conclusions that are even more unexpected.

The next step would be to make the theory more comprehensive. Several theories may be related to CLT such as the *Information Processing Theory* by G. Miller (Miller, 1956), *Human Memory Theory* by A.D. Baddeley, *Cognitive Principles of Multimedia Learning* by R. Moreno and R.E. Mayer (Moreno, 1999), *Parallel Instruction Theory* by R. Min (Min, 1992), *Anchored Instruction* by J.D. Bransford (Bransford, 1990), and *Social Development Theory* by J.V. Wertsch (Wertsch, 1985), among others. All of the relevant theories could be brought under one theory that provides the first principles from which all others are derived. Then, all validations further not only the specific research but provide greater confidence in the theory that is founded on the first principles. Such unification would provide a basis by which the unexpected may actually be determined rather than describing that which is already recognized.

Once again, we have a challenge for Learning Theorists—devise the umbrella theory under which all of the above cited theories can be brought by providing the first principles, basic assumptions upon which all theories rely.

Instruction Theory

Another example for converting a descriptive theory to an axiomatic theory comes from M. David Merrill, "First Principles of Instruction"[33].

Merrill presents five hypotheses that he refers to as *first principles* of learning and asserts a premise that these principles "are necessary for effective and efficient instruction"[34]. Further:

> If this premise is true, there will be a decrement in learning and performance when a given instructional program or practice violates or fails to implement one or more of these first principles.[35]

He continues:

These five first principles stated in their most concise form are as follows:
1. Learning is promoted when learners are engaged in solving real-world problems.
2. Learning is promoted when existing knowledge is activated as a foundation for new knowledge.
3. Learning is promoted when new knowledge is demonstrated to the learner.
4. Learning is promoted when new knowledge is applied by the learner.
5. Learning is promoted when new knowledge is integrated into the learner's world.

These first principles are already well-stated as axioms with but minor modifications. In fact, he also has already provided corollaries for these hypotheses. The conversion to axioms is accomplished as follows:

1. If students are engaged in solving real-world problems, then student learning will increase.
2. If a student uses existing knowledge when learning new knowledge, then student learning will increase.
3. If new knowledge is demonstrated for the student, then student learning will increase.
4. If a student applies new knowledge, then student learning will increase.
5. If a student integrates new knowledge with existing student knowledge, then student learning will increase.

Whereas these first principles are easily converted to the form of axioms, these axioms do not provide a basis for a theory. The reason is that they all have the same conclusion, thus not providing any means to relate the axioms. What this then tells us is that these axioms are actually *prescriptions for learning* that have been derived from a more robust theory—a learning theory. The antecedents of the implications provide the *prescriptions for learning* as follows:

- Solve real-world problems
- Use existing knowledge
- Demonstrate new knowledge
- Apply new knowledge
- Integrate new knowledge with existing knowledge

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[33] Merrill, M. David, (2002). "First Principles of Instruction," ETR&D, Vol. 50, No. 3, pp. 43-59, ISSN 1042-1629.
[34] Ibid., p. 44.
[35] Ibid.

As stated, however, these prescriptions for learning but describe events or empirical observations that can be tested, rather than the foundation for a theory. Rather than an *Instruction Theory*, we have an *Instruction Prescription* that may be of great value to teachers.

If these prescriptions are to be derived from theory, then some or all of these antecedents would have to be the conclusions of other axioms. In fact, these *axioms* would probably be *theorems* within the more robust theory. This then reverts back to the challenge presented earlier:

> Is there a Learning Theorist who can develop the basic principles, the first principles in fact, that will encompass not only the other learning hypotheses cited, but now these principles that seem to have strong support for being valid guidelines that will improve students' learning?

Now we can return to our earlier questions.

What is a 'Theory Model'?

'**Theory model**' will be explicated below in the section on *Theory Development*.

What is an 'Intentional System'?

What is meant by 'Intentional'?

'**Intentional**' is used here to describe a person or group of people who have specific goals which their actions are designed to achieve.

For example, an education system has specific goals, the education of children, that it is designed to achieve.

A military system has specific goals, the protection of a society, that it is designed to achieve.

A person has specific work-goals that he/she "intend" to achieve.

These are all examples of a person or group of people who are *intentional*; that is, they are "goal-oriented".

What is a 'System'?

'**System**' is the connection of objects; for example, people, that are considered to be functioning as a unit.

Each of the above three examples are *systems*—an education system, a military system, and an individual person-system.

Formally, 'System', $\mathbb{S}$, is defined as the following ordered-pair:

$\mathbb{S} =_{Df} (\mathbb{S}_0, \mathbb{S}_R)$, where $\mathbb{S}_0$ is the *object-set* and $\mathbb{S}_R$ is the *relation-set*.

However, this definition of 'system' will be refined later in this study.

We also now see that there are additional terms that have been added that we will have to understand if we are to proceed in a clear manner that avoids confusion; for example, 'functioning' and 'unit'. However, such an endeavor will result in an unending sequence of terms or will result in a circularity of our definitions.

As a result, it will be necessary to start with certain undefined terms that it must be presumed all readers of this study will comprehend. And, it must also be presumed that all readers of this study will have a certain basic background in order to fully comprehend this study.

What Basic Background is Required to Understand this Study?

It is presumed that all readers will be able to follow the development of the basic logic that is used; e.g., the *Sentential Calculus* and *Predicate Calculus*, among others.

There are numerous textbooks on these subjects where the basic knowledge can be acquired, although, to a certain extent, a development of the logics will also be included in this study so that all readers will be informed of what is required.

It is not presumed that a reader has a knowledge of axiomatic theories, since that is one goal of this study—to discuss the nature of and value of axiomatic theories, to provide the rationale for axiomatic theories and why they are necessary for any definitive empirical study where individual-predictive behavior is desired. While today, practically all studies in the social sciences, if not all, rely on statistical-based analyses, such analyses are only, and can only be, group-predictive. If individually-predictive outcomes are desired, then an axiomatic-based analysis **must** be utilized.

Further, it is presumed that any serious reader will obtain and read the seminal work by Elizabeth Steiner: *Methodology of Theory Building*.[36]

Also, the work on APT developed by Theodore Frick, "*Analysis of Patterns in Time (APT)*",[37] will be required as it provides part of the logical basis for *ATIS*:

What Undefined Terms are Required?

The required undefined terms will be presented as necessary during this study. For now, we start with the following undefined terms:

'Intentional', 'Object' (which may also be referred to as 'Element'), and 'Empirical'.

Therefore, what is ATIS?

As previously stated:

ATIS is an *axiomatic formal empirical theory* designed specifically for *intentional systems*.

ATIS is an empirical theory that is designed so that selected parameters can be evaluated to determine projected outcomes in view of these parameters.

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[36] Steiner, Elizabeth, (1988), *Methodology of Theory Building*, Educology Research Associates, Sydney, NSW, Australia.

[37] Frick, T. (1990). "Analysis of Patterns in Time (APT): A Method of Recording and Quantifying Temporal Relations in Education." *American Educational Research Journal*, 27(1).

Applications of *ATIS*

One of the main features of *ATIS* is that it can be used as the basic logic for computer simulations; for example, *SimEd*, *SimTIE*, as designed and developed by Theodore W. Frick, Associate Professor and Web Designer, Indiana University, and other such programs. One program, in particular, that will utilize *ATIS* is SimTIE (*Simulated Totally-Integrated Education*). Thus, *SimTIE* will have an axiomatic empirical theory for its logic which will provide empirical-based predictive outcomes. That is the advantage of *ATIS*.

The outcomes of computer simulations, or computer models, are dependent on the program designed to analyze the selected input parameters. There are essentially two types of programs that can be used for computer models—*Scenario-Based* and *Logic-Based*. Most often, and especially for the "Sim" models, a *Scenario-Based Model* is used. Such models are dependent on the imagination of the designer and comprehensiveness of the data included in the program.

Scenario-Based Models

Scenario-Based Models are defined as programs that provide *scripts* to determine outcomes. The scripts can be *narrative* or *quantitative*.

Narrative scripts characterize the *qualitative parameters* of a system; that is, the *social*, *philosophical*, and *individual* parameters and their *uncertainty of future outcomes*. That is, the predictiveness of the model is pre-determined by the programmer.

Quantitative scripts define the *scientific facts*, known or credible data, and *quantitative models* that are used to *determine future outcomes*. Again, the predictiveness of the model is pre-determined by the programmer in terms of what scientific facts are included in the model.

However, regardless of the type of script, their content is closed; that is, there are a limited number of possible outcomes, and the scripts predetermine the outcomes. Ted Friedman recognizes this closed characteristic of *Scenario-Based Models* in his report on SimCity, "Semiotics of SimCity,"[38] when he states:

> Of course, however much "freedom" computer game designers grant players, any simulation will be rooted in a set of baseline assumptions. SimCity has been criticized from both the left and right for its economic model. It assumes that low taxes will encourage growth while high taxes will hasten recessions. It discourages nuclear power, while rewarding investment in mass transit. And most fundamentally, it rests on the empiricist, technophilic fantasy that the complex dynamics of city development can be abstracted, quantified, simulated, and micromanaged.

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[38] "Semiotics of SimCity," *First Monday*, Peer-Reviewed Journal on the Internet, http://www.firstmonday.dk/issues/issue4_4/friedman/.

Logic-Based Models

On the other hand, *Logic-Based Models* are not dependent on analyses of predetermined values, but on the logic of a theory that has been shown to be valid for the target empirical system; for example, an education system. The theory describes the *empirical system* in terms of its affect relations, properties, and axioms. The theory logic is then used to project outcomes founded on the theory with respect to input parameters. The value of the *Logic-Based Model* is then determined by the skill of the designer to provide an appropriate and accurate program that corresponds to the input. Designed properly, a *Logic-Based Model* should be able to provide projected outcomes that are open-ended.

Unlike *Scenario-Based Models* that are closed due to the limited number of scripts, *Logic-Based Models* have an infinite number of outcomes.

This is especially so with the logic designed for the *ATIS Model*. Due to the number of axioms involved, over 200, there are initially tens-of-thousands of theorems that can be obtained. However, the SCTs (*Structural Construction Theorems*) provide for an open-ended and infinite number of additional theorems. The reason is that new affect relations, properties, or system-descriptive parameters can be inserted into the SCTs that will automatically generate thousands of additional theorems. The *Logic-Based Model* is not dependent on what has been initially programmed for the logic, but what is subsequently programmed as a result of new system parameters.

The strength of a *Logic-Based Model* will be seen in what follows, and additional analyses relating to the two types of models will be discussed.

Theory Development

Elizabeth Steiner has explicated the concepts of 'theory' and 'model' in great detail. She has also explicated the relation between 'theory' and 'model'. We will start with her analyses.[39]

It is commonly believed that developing theory is derived from one of two logical processes—*induction* or *deduction*. Theory development, however, is not derived from either.

Induction brings together many observations from which it is claimed a generalization, or pattern, can be obtained. Such, however, is not the case.

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[39] See *Methodology of Theory Building*, Elizabeth Steiner, Indiana University, Educology Research Associates, Sydney, 1988. This text is an extension of her earlier work that includes the following: *The Model in Theorizing and Research*, Elizabeth Steiner Maccia, Educational Theory Center, The Ohio State University, May, 1965; *Sources of Empirical Theory*, Elizabeth Steiner Maccia, The Ohio State University, May, 1964; *Theory Construction and Models*, Elizabeth Steiner Maccia, The Ohio State University, May, 1964; *The Conceptions of Model in Educational Theorizing*, Elizabeth Steiner Maccia, Center for the Construction of Theory in Education, The Ohio State University, August, 1962; *Models and the Meaning of 'Retroduction'*, Elizabeth Steiner Maccia, Foundations Division, The Ohio State University, June, 1962; and *The Way of Educational Theorizing Through Models*, Elizabeth Steiner Maccia and George S. Maccia, Foundations Division, The Ohio State University, June, 1962.

It is clear that in order to obtain the pattern, the observer must be able to recognize the pattern, and therefore has brought the generalization to the data. The data does not induce the generalization, the observer does.

This is a mode of inquiry, but it is not theorizing. Today induction provides the basis for the data-mining technologies that are widely used to develop structure from unstructured data. The structuring of unstructured data is not theorizing. Induction is a means of evaluating data so as to recognize and develop patterns. The patterns, when logically organized, are the theory that the data confirms by induction.

Deduction is a means of explicating existing theory; that is, "to clarify and complete theory" [Steiner, *Models and the Meaning of 'Retroduction'*]. The theory presents the postulates or axioms upon which the theory is based. Deduction then provides the logical process by which the theory is made explicit through its deductive statements. Deduction generates conclusions of the theory in the form of theorems or hypotheses that are to be evaluated for validity.

Deduction explicates a theory into statements, and *induction* evaluates the statements.

What then is the logical process by which theory is developed? *Retroduction*.

Retroduction

'*Retroduction*' is a "moving backward." From one perspective we move backward to devise another perspective.

For example, the "*Holographic Paradigm*" may provide a perspective for education. Considering that a holograph can be generated from any of its facets, it may suggest that a student may learn, not by focusing on the subject of concern directly, but by developing coordinated skills in a discipline not normally considered as being directly relevant. But, the "parts" of the divergent discipline may have "facets" that in fact produce the entire "hologram" in the subject of concern. For example, one may learn how to interpret historical events by taking acting lessons whereby the skill is developed that allows one to become immersed in a period thinking and lifestyle.

In terms of theory development, one theory that can be used as a model for developing another theory is a "devising theory."

Retroduction is the process of using one theory as a model to devise another theory. Therefore:

(1) *Retroduction* devises theory,
(2) *Deduction* explicates theory, and
(3) *Induction* evaluates theory.

'*Retroduction'* and '*Abduction'* Confusion

There is a prevailing misconception concerning '*retroduction'* and '*abduction'*.

Retroduction is normally, if not universally, defined as *abduction*. Such a definition is in error. First, there is a recognizable distinction between a "taking from," *abduction*, and a "moving backward," *retroduction*.

It is presumed that Peirce generally defines 'abduction' and 'retroduction' as the same, although a careful reading indicates that he does not. An analysis of this confusion is worth considering due to its almost universal acceptance.

The *'retroduction'* and *'abduction'* confusion seems to have come from the work edited by Charles Hartshorne and Paul Weiss, *Collected Papers of Charles Sanders Peirce*.[40] For example, if one goes to the index for Volume 1, the reference for 'retroduction' is: "*see* Abduction." The implication is that they are the same. But, when we look at the first reference for 'abduction', §65, we find that the two are not the same at all. Peirce writes:

> **§10. KINDS OF REASONING**
> 65.   There are in science three fundamentally different kinds of reasoning, Deduction (called by Aristotle συναγωγή or αναγωγή), Induction (Aristotle's and Plato's ἐπαγωγή) and Retroduction (Aristotle's ἁπαγωγή), but misunderstood because of corrupt text, and as misunderstood usually translated *abduction*. Besides these three, Analogy (Aristotle's παραδειγμα) combines the characters of Induction and Retroduction.

It should be clear that 'retroduction' and 'abduction' are not the same, and that they have been equated only because of "corrupted text."

So, what is the distinction between 'retroduction' and 'abduction'? Consider the following examples.

> A *direct affect relation and its measure* in a behavioral topological space are defined in terms of a *mathematical vector*. That is, it is recognized that the concept of "vector" is applicable to this behavioral theory. This transition was recognized as a result of affect relations being interpreted as "force fields."

> Gravitational and electromagnetic force fields are vector fields; fluid velocity vectors, whether in the ocean or the atmosphere, are vector fields; and weather pressure gradients are vector fields. Affect relations within a behavioral system are vector fields—they are dynamic. They exhibit both direction and magnitude. They exhibit the change and flow of any other empirical vector field.

This process of applying an interpretation to the mathematical construct "vector," is a logical process of *abduction*. This is not a process of "moving backward," but a process of "taking from." The mathematical measure is simply being applied to the content of a behavioral theory.

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[40] Hartshorne, Charles and Paul Weiss, *Collected Papers of Charles Sanders Peirce*, The Belknap Press of Harvard University Press, Cambridge (1960).

There is no theory development; there is simply an interpretation of the theory by mathematical means. The mathematical concept of a vector field is utilized as a measure to further explicate the theory. The affect relation concept was already in the theory, so it is clear that no theory development was accomplished. Was there a retroduction of the "form" as a single predicate from mathematics? No. What is being utilized here is simply the definition of a vector field. The definition of "vector field" is being "taken from" mathematics in order to deductively explicate the theory of affect relations. As Thompson remarked following the classification of the SIGGS properties by Frick: "It was recognized that the Structural Properties represented the topology of the theory." Such recognition was a deductive process and not a retroductive process. The mathematical vector field theory was not used to devise the SIGGS theory; it was used to explicate the SIGGS theory. (SIGGS: Set theory, Information theory, Graph theory, General Systems theory.)

This study defines 'retroduction' and 'abduction' as distinct logical processes, and such that they complement the logical process of 'deduction,' as follows:

- **Deduction** is the logical process by which a conclusion is obtained as the implication of assumptions.
- **Retroduction** is the logical process by which a point of view is utilized to devise a conjecture or theory.
- **Abduction** is the logical process by which a theoretical construct of one theory is utilized to analyze or interpret the parameters of another theory.

These distinctions are formalized below.

While the *Deduction Theorem* is a standard part of mathematic logic, this study extends this analysis to include the *Retroduction Theorem* and *Abduction Theorem*.

Deduction Theorem. *The* Deduction Theorem *will be stated first. The applicable logical schema of the Sentential Calculus is:*

$$\text{If } \mathcal{P} \supset \mathcal{Q}, \text{ then } \mathcal{P} \vdash \mathcal{Q}; \text{ and If } \mathcal{P} \vdash \mathcal{Q}, \text{ then } \mathcal{P} \supset \mathcal{Q} \mathrel{..\equiv..} \mathcal{P} \vdash \mathcal{Q} \mathrel{.\equiv.} \vdash \mathcal{P} \supset \mathcal{Q}$$

The *Deduction Theorem* is a statement of the following implication:

$$\mathcal{P} \vdash \mathcal{Q} \mathrel{.\supset.} \vdash \mathcal{P} \supset \mathcal{Q}$$

The statement of the *Retroduction Theorem* and *Abduction Theorem* are much more complex.

Retroduction Theorem. *We will first take a look at the concept of* **Retroduction** *as defined by Steiner:*

> Given theories $\mathcal{A}$ and $\mathcal{B}$, theory $\mathcal{A}$ is a devising model for theory $\mathcal{B}$ if there is a subset, $\mathcal{C}$, of $\mathcal{A}$ such that the predicates of $\mathcal{B}$ are a representation in substance or form of the predicates of $\mathcal{C}$; whatever is true of $\mathcal{C}$ is true of $\mathcal{B}$; and not whatever is true of $\mathcal{B}$ is true of $\mathcal{A}$.

Initially it would appear that the following implication holds: $\mathcal{A} \supset \mathcal{B}$. However, as Steiner points out, "The theory or conjecture that emerges (conclusion) contains more than the theory or point of view from which it emerges (premises). The implication, then, can only hold from the conclusion to the premise"; that is, $\mathcal{B} \supset \mathcal{A}$.   It could be argued that the sentential and predicate logic do not hold in this instance. But, if not, we are left with a state of confusion when we are attempting to develop a scientific theory that relies on just such logics. Therefore, it must be assumed that the logic holds and we need to take a closer look at just what is required.

Taking retroduction, as it is conceptually defined; we have that Theory $\mathcal{A}$ is a devising model for Theory $\mathcal{B}$. By this is meant that the predicates for Theory $\mathcal{B}$ are derived as representations from a subset, $\mathcal{C}$, of the predicates of Theory $\mathcal{A}$; that is, $\mathcal{P}(\hat{h}) \in \mathcal{B} \supset \mathcal{P}(h) \in \mathcal{C} \subset \mathcal{A}$.

But also we have that Theory $\mathcal{B}$ results in more than what was in Theory $\mathcal{A}$; since, otherwise, it would not be an emendation of Theory $\mathcal{A}$, but simply a replication. This emendation of Theory $\mathcal{A}$ that results in more than what is in $\mathcal{A}$ is formally defined as:

$$\exists \mathcal{P}(\hat{h}) \in \mathcal{B}[\forall \mathcal{P}(h) \in \mathcal{A}[\sim(\mathcal{P}(\hat{h}) \in \mathcal{B} \doteq \mathcal{P}(h) \in \mathcal{A}];$$

> where '∃' is read "there exists", '∼' is read "not" or "it is not the case that", '∀' is read "for all", and '≐' is read "is isostruct to"; and isostructism is a mapping of one entity to another to which it is isomorphic or isosubstantive. (That is, "There exists a Predicate $\hat{h}$, an element of $\mathcal{B}$, such that, it is not the case that the Predicate $\hat{h}$ is an element of $\mathcal{B}$ is isostruct to Predicate h an element of $\mathcal{A}$.) This meets the final requirement by Steiner.

Further, it is required that the "predicates of $\mathcal{B}$ are a representation in substance or form of the predicates of $\mathcal{C}$." This requirement is formalized as follows:

$$P(h) \in \mathcal{C} \overset{?}{\doteq} P(\hat{h}) \in \mathcal{B} =_{df} P(h) \in \mathcal{C} \cong P(\hat{h}) \in \mathcal{B} :\vee: P(h) \in \mathcal{C} \cong P(\hat{h}) \in \mathcal{B};$$

> where, '≅' =$_{df}$ "is isomorphic to" and '≅' =$_{df}$ "is isosubstantive to"; where '=$_{df}$' is read "is defined as".

An isostructism is a mapping of one entity to another to which it is isomorphic or isosubstantive. An isomorphism is a mapping of one entity into another having the same elemental structure, whereby the behaviors of the two entities are identically describable by their affect relations. An isosubstantism is a mapping of one entity into another having similar predicate descriptors.

Therefore, all of Steiner's stipulations have been met. As a result, the *Retroduction Theorem* is formalized as follows:

$$\exists \mathcal{P}(\hat{h}) \in \mathcal{B} [\forall \mathcal{P}(h) \in _c\mathcal{A}[\sim(\mathcal{P}(\hat{h}) \in \mathcal{B} \stackrel{.}{=} \mathcal{P}(h) \in _c\mathcal{A}],$$
$$\mathcal{P}(h) \in \mathcal{C} \subset _c\mathcal{A} \stackrel{?}{=} \mathcal{P}(\hat{h}) \in \mathcal{B},$$
$$\exists \mathcal{P}(\hat{h}) \in \mathcal{B}[\sim(\mathcal{P}(\hat{h}) \in \mathcal{B} \stackrel{?}{=} \mathcal{P}(h) \in _c\mathcal{A})] \vdash$$
$$\mathcal{P}(\hat{h}) \in \mathcal{B} \supset \mathcal{P}(h) \in \mathcal{C} \subset _c\mathcal{A}.$$

Proof of Retroduction Theorem:

For the purposes of this proof, since the conclusion is simply the result of the assumptions by definition, all that needs to be argued is that the *Predicate Calculus* applies to theory $\mathcal{B}$. To apply, theories $_c\mathcal{A}$ and $\mathcal{B}$ must be isostruct with respect to $\mathcal{C}$. By assumption, they are.   All we have to show is that $\mathcal{P}(\hat{h}) \in \mathcal{B}$ represents a consistent set of predicates that have been derived from Theory $_c\mathcal{A}$ and that they make $\mathcal{B}$ a theory. The formal proof is:

$\mathcal{P}(h) \in _c\mathcal{A}$	Assumption
$\mathcal{P}(\hat{h}) \in \mathcal{B} \supset \mathcal{P}(h) \in \mathcal{C} \subset _c\mathcal{A}$	Assumption
$\exists \mathcal{P}(\hat{h}) \in \mathcal{B} \ \forall \mathcal{P}(h) \in A[\sim(\mathcal{P}(\hat{h}) \supset \mathcal{P}(h))]$	Assumption
$\mathcal{P}(\hat{h}) \in \mathcal{B}$ are derived from $\mathcal{P}(h) \in \mathcal{C} \subset _c\mathcal{A}$	Assumption

All we now need to demonstrate is that $\mathcal{P}(\hat{h}) \in \mathcal{B}$ is a consistent theory.

If $\{(\hat{w},\hat{y}) \in \mathcal{B} \times \mathcal{B} \mid \mathcal{P}(\hat{w},\hat{y})\} \cong \{(w,y) \in _c\mathcal{A} \times _c\mathcal{A} \mid \mathcal{P}(w,y)\}$, then all of the consistent logical conclusions relating to $\mathcal{P}(w,y)$ also apply to $\mathcal{P}(\hat{w},\hat{y})$, by substitution.

If $\mathcal{P}(\hat{h}) \cong \mathcal{P}(h)$, then any component of $_c\mathcal{A}$ that satisfies $\mathcal{P}(h)$ has a corresponding component in $\mathcal{B}$ that satisfies $\mathcal{P}(\hat{h})$.

Therefore, the components, relations, and predicates which are valid for Theory $_c\mathcal{A}$ have corresponding components, relations, and predicates in $\mathcal{B}$, resulting in the consistency of $\mathcal{B}$. By definition, the predicates of $\mathcal{B}$ comprise a theory.

Since the theories are isostruct, any proof in $\mathcal{C}$ is applicable to a corresponding proof in $\mathcal{B}$, since they will have corresponding axioms and assumptions. Further, any predicate in $\mathcal{B}$ not in $_c\mathcal{A}$ can be taken as an assumption or axiom from which resulting theorems can be derived by the *Sentential* and *Predicate Calculi*.

The value of this theorem is that it establishes that the logic of the *Axiomatic Sentential* and *Predicate Calculi* apply to theory $\mathcal{B}$.

As has been shown above, there is a distinction between *retroduction* and *abduction*. The *Abduction Theorem* is given below.

<u>Abduction Theorem</u>. *Given theories $\mathcal{A}$ and $\mathcal{B}$, theory $\mathcal{A}$ is a formal model-of theory $\mathcal{B}$ if there is a subset, $\mathcal{C}$, of $\mathcal{A}$ such that the predicates of $\mathcal{B}$ are an equivalent representation in form of the predicates of $\mathcal{C}$; whatever is true of $\mathcal{C}$ is true of $\mathcal{B}$; and whatever is true of $\mathcal{B}$ is true of $\mathcal{C}$.*

The formal statement of the Abduction Theorem is:

$$\mathrm{h} \equiv \hat{\mathrm{h}}, \; \mathcal{P}(\mathrm{h}) \approx \mathcal{P}(\hat{\mathrm{h}}), \; \mathcal{P}(\mathrm{h}) \approx \mathcal{P}(\hat{\mathrm{h}}) \vdash \mathcal{P}(\mathrm{h}) \in \mathcal{C} \subset \mathcal{A} :\equiv: \mathcal{P}(\hat{\mathrm{h}}) \in \mathcal{G} \subset \mathcal{B}$$

Proof of Abduction Theorem:

(1) $\mathrm{h} \equiv \hat{\mathrm{h}}$	Assumption
(2) $\mathcal{P}(\mathrm{h}) \approx \mathcal{P}(\hat{\mathrm{h}})$	Assumption
(3) $\mathcal{P}(\mathrm{h}_1), \mathcal{P}(\mathrm{h}_2), …, \mathcal{P}(\mathrm{h}_n) \in \mathcal{C} \subset \mathcal{A}$	Assumption
(4) $\mathcal{P}(\hat{\mathrm{h}}_1), \mathcal{P}(\hat{\mathrm{h}}_2), …, \mathcal{P}(\hat{\mathrm{h}}_n) \in \mathcal{G} \subset \mathcal{B}$	Substitution, 1 in 3
(5) $\therefore \mathcal{P}(\mathrm{h}_1), \mathcal{P}(\mathrm{h}_2), …, \mathcal{P}(\mathrm{h}_n) \in \mathcal{C} \vdash \mathcal{P}(\hat{\mathrm{h}}_1), \mathcal{P}(\hat{\mathrm{h}}_2), …, \mathcal{P}(\hat{\mathrm{h}}_n) \in \mathcal{G}$	From 3 and 4
(6) $\vdash \mathcal{P}(\mathrm{h}_1), \mathcal{P}(\mathrm{h}_2), …, \mathcal{P}(\mathrm{h}_n) \in \mathcal{C} \supset \mathcal{P}(\hat{\mathrm{h}}_1), \mathcal{P}(\hat{\mathrm{h}}_2), …, \mathcal{P}(\hat{\mathrm{h}}_n) \in \mathcal{G}$	Deduction Theorem on 5
(7) $\therefore \mathcal{P}(\hat{\mathrm{h}}_1), \mathcal{P}(\hat{\mathrm{h}}_2), …, \mathcal{P}(\hat{\mathrm{h}}_n) \in \mathcal{B} \vdash \mathcal{P}(\mathrm{h}_1), \mathcal{P}(\mathrm{h}_2), …, \mathcal{P}(\mathrm{h}_n) \in \mathcal{C} \subset \mathcal{A}$	From 4 and 3
(8) $\vdash \mathcal{P}(\hat{\mathrm{h}}_1), \mathcal{P}(\hat{\mathrm{h}}_2), …, \mathcal{P}(\hat{\mathrm{h}}_n) \in \mathcal{B} \supset \mathcal{P}(\mathrm{h}_1), \mathcal{P}(\mathrm{h}_2), …, \mathcal{P}(\mathrm{h}_n) \in \mathcal{C} \subset \mathcal{A}$	Deduction Theorem on 7
(9) $\mathcal{P}(\mathrm{h}) \approx \mathcal{P}(\hat{\mathrm{h}}) \vdash \mathcal{P}(\mathrm{h}_1), …, \mathcal{P}(\mathrm{h}_n) \in \mathcal{C} \subset \mathcal{A} :\equiv: \mathcal{P}(\hat{\mathrm{h}}_1), …, \mathcal{P}(\hat{\mathrm{h}}_n) \in \mathcal{B}$	Definition, 4 & 7 / Q.E.D.

The significance of this theorem is that formal predicates of a given theory that are isomorphic to formal predicates of another theory, define the properties of the second theory.

Types of Models

Steiner presents the concept of 'model' as a dichotomy: '*model-of*' and '*model-for*'.

Intuitively, '*model-of*' corresponds to the familiar type of construction models—model cars, model planes, etc. Also, intuitively, '*model-for*' corresponds to the familiar type of exemplary models—professional models who exemplify appearances or role models who exemplify behaviors.

From these examples, it is seen that a '*model-of*' is a representation of an object, possibly a "scale model"; and a '*model-for*' is the object that is being represented in an ideal; for example, a "super model."

> *Model-of* is a scaled version of the intended object.

> *Model-for* is a paradigm that can be used to describe ideal structures.

These models are designated, respectively, *first-order model* and *second-order model*.

Models that are used to devise theory by retroduction are *second-order models*. They provide the perspective desired to develop the new theory. Thus Steiner defines the relation between the types of logic and the types of models as follows:

> **Retroductive Logic** $=_{df}$ Devising of theory from a *second-order model*.

> **Deductive Logic** $=_{df}$ Explicating a theory for clarification or completeness.

> **Inductive Logic** $=_{df}$ Evaluating a theory to delineate the range of defined objects.

A theory may be further delineated by the referents of the theory. If the theory is about actually existing objects, then it will be called an '*empirical theory*'. For example, theorizing about social referents is an attempt to characterize actually existing objects falling within the domain of some social context or process. Education theorizing is such a theory; it is empirical theorizing. In an *empirical theory*, the statements not only express the nature of the objects, but also the way in which the objects are interrelated.

In view of the preceding, in this study, *ATIS* is a *model-for* education theorizing. *SimEd* or *SimTIE*, on the other hand, are *models-of* educational systems.

Now that the type of theorizing has been established, the *ATIS* model will be further explicated. *ATIS* is a logico-mathematical model; that is, it is a *formal model* with logical and mathematical formalizations.

ATIS as a Mathematical Model Theory

In mathematics, model theory is defined as a branch of logic that studies mathematical structure, and, in particular, the structures of axiomatic set theory. *ATIS* is a generalization of mathematical model theory.

Axiomatic set theory is set theory founded on axioms with no empirical content. As set theory is closely associated with mathematical logic, there is an integration of the *Sentential* and *Predicate Calculi* in *ATIS* that results in a formal theory that provides the rigor of deduction and proof.

While the properties and axioms of *ATIS* are initially framed in the context of an empirical theory, those properties and axioms are transformed into a formal logico-mathematical theory that allows for the analysis of *ATIS* as a formal theory.

Mesarović's General Systems Theory Mathematical Model. Others have developed mathematical models for general systems theory. One in particular, Mihajlo D. Mesarović, has developed this area extensively.

Mihajlo D. Mesarović, in "A Mathematical Theory of General Systems,"[41] has developed measures for system properties. In his work, Mesarović restricts the measures to "General Systems Theory of Hierarchical Systems."[42] The mathematical measures developed by Thompson in this study are a generalization of the Mesarović measures as extended by Yi Lin.[43]

However, Mesarović also introduces a "coordination strategy" that will not be applied to *ATIS* measures. This strategy was designed by Mesarović to "adjust" the theoretical projections with actual observations. As described, it appears to simply classify two sets of systems, those that can be "adjusted" and those that cannot. Such a dichotomy is not appropriate for the type of systems here being considered. For *ATIS*, the criteria for verification are with respect to the theorems of the theory without adjustment.

The distinction between the Mesarović approach and that proposed here is that Mesarović relies on models that are "scientifically" developed yet closed, whereas the approach here is founded on the logic of system's theory; such logic resulting in an open-ended theory that provides for an infinite number of outcomes.

As described earlier, the proposed model for this research can be tailored to the specific needs of an empirical system without having to modify the initial program, as would have to be done in a *Scenario-Based Model*.

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[41] Mesarović, Mihajlo D. (1972), "A Mathematical Theory of General Systems," *Trends in General Systems Theory*, Ed., George J. Klir, Wiley-Interscience, p. 251.

[42] Ibid., p. 264.

[43] Lin, Yi (1999), *General Systems Theory: A Mathematical Approach*, Kluwer Academic/Plenum Publishers, NY.

While Mesarović has contributed greatly to the mathematical development of general systems theory, his system models do not have a basis founded on theory. One such model is WIM (World Integrated Model) that was developed with 49 subroutines. It was quite refined in that it utilized about 21,000 numbers to describe the state of the global system at any one time. At one time, WIM could be viewed online at http://genie.cwru.edu/scenarioanalysis.htm; however, it is no longer available.

The direction being taken by the study herein presented is distinct from that of most, if not all, other social models—this *SimEd Model* will rely on a *Logic-Based Model* for its projections. It is believed that the parameters are too numerous and the possible outcomes are so extensive that anything less would result in a model that could end up with the same shortcomings as that recognized by Friedman concerning SimCity.

With this introduction to *ATIS*, it is seen that in order to have a legitimate empirical model that can result in empirically-verifiable empirical results, the model must be founded on a Logic-Based Model, and that *ATIS*, being an axiomatic theory, provides just such a basis.

We will now take a more in-depth look at the development of *ATIS* and its value for empirical studies of intentional systems.

'*Theory*' and '*General System*' Definitions

The definitions of *theory* and *General System* must be considered in more detail, and how they can address the needs of those industries concerned with predicting system outcomes founded on various structural scenarios desired by an organization or as the result of empirical observations.

Since the systems of concern are characterized by vast amounts of information, we can come to an understanding of such systems by seeing what is being done in industries that have dealt with large amounts of information. One such industry that must interpret vast amounts of data is the counter-terrorist industry. This industry previously utilized the *Total Information Awareness (TIA) System*, but it was discontinued due to privacy concerns. However, since it utilized data-mining technologies, it would have been limited in its ability to actually provide individually-predictive outcomes.

A report concerning the *TIA System*, however, does highlight the direction being pursued in this industry to respond to the terrorist threat, and provides key points that should be considered when attempting to find a means by which the terrorist threat can be identified.

In a report by the *Defense Advanced Research Projects Agency's Information Awareness Office*, it states:

> "It is difficult to counter the threat that terrorists pose. Currently, terrorists are able to move freely throughout the world, to hide when necessary, to find unpunished sponsorship and support, to operate in small, independent cells, and to strike infrequently, exploiting weapons of mass effects and media response to influence governments. This low-intensity/low-density form of warfare has an information signature, albeit not one that our intelligence infrastructure and other government agencies are optimized to detect. **In all cases, terrorists have left detectable clues that are generally found after an attack**."

Therefore, what is needed is a means of identifying the "clues" **prior** to an attack. It is also noted that similar concerns are confronted in our pursuit of a reliable education system where it is desirable to be able to discern outcomes of changes in education methodologies in a timely manner rather than having to wait 12 or more years before results can be recognized.

In response to the terrorist threat, the technologies being sought are characterized as follows:

➢ Real time learning, pattern-matching and anomalous pattern detection
➢ Human network analysis and behavior model building engines
➢ Event prediction and capability development model building engines
➢ Data mining of unstructured data
➢ Information discovery through statistical methods

As noted, these same concerns can be applied to education systems.

The following charts depict the efforts that the TIA was trying to address, and can be found at the following website:
(http://en.wikipedia.org/wiki/Information_Awareness_Office#Components_of_TIA_projects_that_continue_to_be_developed)
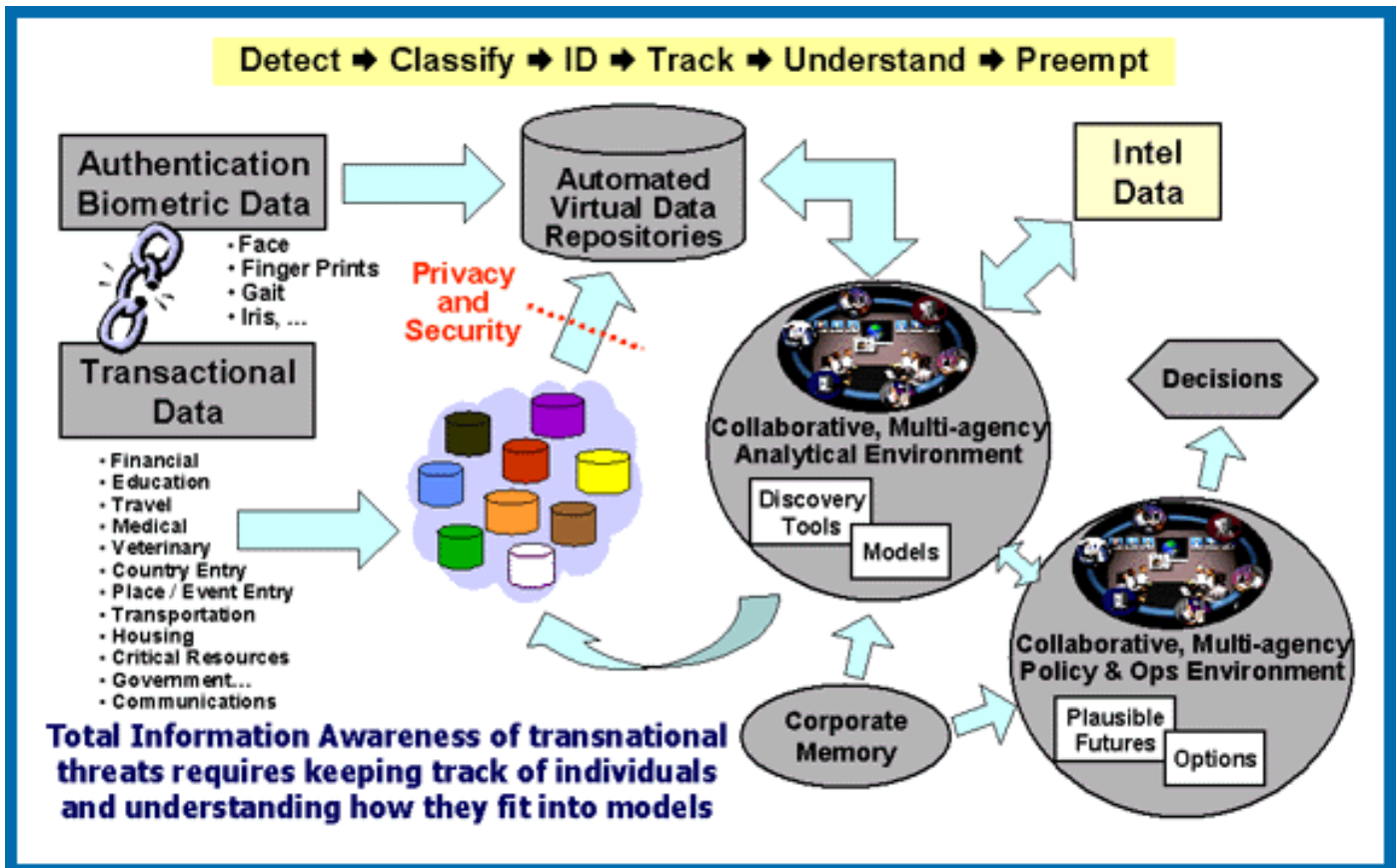
**Diagram of *Total Information Awareness System*, taken from official
(decommissioned) Information Awareness Office website**

What this chart does show is the vast amount of information that must be integrated and analyzed in order to determine predictive outcomes.

However, the difficulty with all of the approaches cited above, the technologies being sought, is that they rely on statistical-based analyses. The same can be said for any industry today where predictive technologies are being considered; and especially in education. What is actually wanted, however, is the ability to detect *discrete indicators*, not *group indicators* that are provided by statistical-based analyses. That is, "**In all cases, terrorists have left detectable clues that are generally found after an attack.**" These are *discrete indicators* that can never be identified by the use of *group indicators*.

For example, in education, we can frequently recognize problems after the fact and can recognize "indicators" that should have pointed us in the right direction. For example, on-going modifications of a school system or a class are done in response to "feedback" that the current plan needs modification. Those indicators are discrete and in order to optimize instruction, they must be found *prior* to the compromising of instruction. Statistical-based analyses, *by design*, <u>**cannot**</u> detect such indicators, since they only "detect" **that which has already happened** and only with respect to the group and not the individual.

When considering how to predict system optimization, we must first understand the nature of the problem and what solutions are even feasible for solving the problem. We will start with the following premise:

Group-Predictive Premise

Statistical analyses are, by design, only group-predictive, and can—*by design*—
never be individually predictive; that is, they can never identify *discrete indicators*.

It is important to state this premise since essentially every approach now being taken in education, business, the military, etc. is founded on a statistical-based analysis. Hypotheses are verified by statistical studies. This is especially the case with respect to unstructured data. *Data-mining* is a critical tool for developing **patterns** of business or educational behavior. However, such analyses **cannot** provide discrete predictions. Further, in all such analyses, patterns must be determined and that can only be done after the activity is well developed.

An axiomatic theory; however, can provide analyses with respect to discrete indicators and is required for predicting individual outcomes.

We will start by taking another look at the definition of *General System* and how it should be defined in order to analyze intentional systems.

Definition of *General System*

From a review of the literature, it is clear that there are various definitions of *system* as well as *general system*. Some of the definitions are required due to mathematical concerns. Others are very imprecise and are used for descriptive arguments rather than logical or mathematical precision. The definition used here follows the convention in mathematics of a system, $\mathbb{S}$, being an ordered pair consisting of an object-set, S, and a relation set, $\mathcal{R}$; that is:

$$\mathbb{S} = (\mathbf{S},\mathcal{R}).$$

This definition can be brought into the context of education or terrorist network systems by citing the definition provided by Steiner and Maccia as follows:

System, $\mathbb{S}$, =$_{df}$ A *group* with at least one affect relation that has information.[44]

$$\mathbb{S} =_{df} (\mathbf{S},\mathcal{R}) = (\mathbb{S}_{x},\mathbb{S}_{\phi}); \text{ where } \mathbf{S} = \mathbb{S}_{x} \text{ and } \mathcal{R} = \mathbb{S}_{\phi}.$$

A **system** is an ordered pair defined by an *object-set*, $\mathbb{S}_{x}$, and a *relation-set*, $\mathbb{S}_{\phi}$.

It is noted that with the development of *ATIS*, the requirement that the affect-relation "has information" has been dropped due to theoretical concerns.

In this study, a more extended definition of system is required to more fully define *General System*. This extension is also required to more clearly define the topology and/or relatedness of a system by its object-sets and relation-sets; as well as allow for a more rigorous and comprehensive development of the system logic required for a logical analysis.

A *General System* is defined within a ***Universe of Discourse***, $\mathcal{U}$, that includes the system and its environment. The only thing that demarcates the systems under consideration is the "Universe of Discourse." And, while that universe may be somewhat fuzzy, whatever systems are being considered will be well defined. In the case of *Educational Systems* the boundary of the universe may be quite fluid, or possibly unknown, especially with respect to the object-sets.

$\mathcal{U}$ is partitioned into two disjoint systems, $\mathbb{S}$ and $\mathbb{S}'$. Therefore, ***Universe of Discourse*** has the following property:

$$\mathcal{U} = \mathbb{S} \cup \mathbb{S}'; \text{ such that, } \mathbb{S} \cap \mathbb{S}' = \emptyset.$$

The disjoint systems of $\mathcal{U}$, $\mathbb{S}$ and $\mathbb{S}'$, are defined as *system* and *negasystem*, respectively.

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[44] Steiner, Elizabeth and Maccia, George, (1966), *Development of Education Theory Derived from Three Educational Theory Models*, Project No. 5-0638, Contract No. OE4-10-186, The Ohio State University Research Foundation, Columbus, Ohio, p. 44.

System environment and *negasystem environment* are defined as follows:

System environment, $\mathfrak{S}'$, =df The components of the universe not in the system.

Negasystem environment, $\mathfrak{S}$, =df The components of the universe not in the negasystem.

A ***General System***, $\mathcal{G}$, is frequently defined by the following parameters:

- *Family of Affect Relations Set*, $\mathcal{A}$;
- *Object Partitioning Set*, $\mathcal{P}$;
- *Transition Function Set*, $\mathcal{T}$;
- *Linearly Ordered Time Set*, $\mathcal{T}$; and
- *System State-Transition Function*, σ.

That is:

General system, $\mathcal{G}$, =df A set of affect relations, partitioned components, transition functions, time set and a system state-transition function.

$$\mathcal{G} =_{df} (\mathcal{A}, \mathcal{P}, \mathcal{T}, \mathcal{T}, \sigma);$$

However, although, this definition of *General System* provides a fairly comprehensive view of what is required to properly analyze the functioning of a system, in order to address the concerns of the desired intentional systems this definition needs to be extended. First, the above cited parameters are defined as follows:

*<u>**Affect Relation Set**</u>: $\mathcal{A}$, the Affect-Relation Set, corresponds to the previously stated relation-set, $\mathfrak{S}_\phi$. Intuitively, this is the set that contains all of the "relations" between elements of $\mathcal{U}$. This is a "set of sets." However, in this case, the sets are defined by specific Affect Relations. For example, these sets will be defined for an educational system as follows: "Teacher-Student Instructional Relation"; "Student-Textbook Relation"; "Student-Parent Relation"; "Administrator-Business Community Relation"; etc. As can be seen, this set can become quite large with numerous subsets, the various relations of an educational system. This is where the concern of "system refinement" must be considered; that is, when selecting relations that are to be considered for the educational system, the <u>**Least-Refined Definition Principle**</u> must be employed.*

<u>**Least-Refined Definition Principle**</u>*: Any system can be viewed with greater refinement, but the level of refinement must be minimized to obtain the greatest predictive results—a view that is possibly counter-intuitive.*

Possibly the easiest way to visualize this is to remember that you do not want to let the minute details get in the way of being able to see what is going on. If we were cognizant of everything around us, we would not be able to function properly due to all the *noise*.

Object Partitioning Set: *$\mathcal{P}$, the* Object Partitioning Set, *corresponds to the previously stated object-set, $\mathcal{S}_x$. Intuitively, this is the set that contains all of the "things" within an educational system or terrorist network system or other large system and its negasystem: students, teachers, administrators, learning materials, community resources, etc.; or terrorists, financial resources, supporters, etc. What is special about this set, however, is that it is a "set of sets"; its elements are subsets of $\mathcal{S}_x$. And, any one object of the system can be in only one subset, hence the name "Partitioning." For example, even if a "student" at times fills an instructional capacity, the individual can only be placed in one set—either the individual is a "student" or a "teacher," but not both* at any one time.

Transition Function Set: *$\mathcal{T}$, the* Transition Function Set, *is necessary in order to "move" objects about the Universe, $\mathcal{U}$. The elements of this set are the functions defined by* feedin, feedstore, input-feedintra, storeput-feedintra, feedout, feedthrough, feedenviron, *and* feedback. *Without them, nothing moves. Also this provides for the dynamics of the system whereby individuals, as above, can be placed in the "student set" at one time and the "teacher set" at another time.*

Linearly Ordered Time Set: *$\mathscr{T}$, the* Linearly Ordered Time Set, *is required in order to give the intentional systems a* dynamic property. *This set helps to keep the system organized! Intuitively, this set may be the easiest one to apply to the educational system. That is, essentially it allows you to attach to an event the appropriate "time" that the event occurs. Without this set there would be no* order *or* sequence *to the events of the system. Also, this is necessary for any application of APT Values (Scores) [*Analysis of Patterns in Time Values*].*[45]

System State-Transition Function: *σ, the* System State-Transition Function, *is required in order to alter the "state" of an educational system. Whereas $\mathcal{T}$, the* Transition Function Set, *moves objects about the system, σ changes the state of the system as a result of the new* Affect-Relations *defined by the move or new affect-relations introduced into the system. Both $\mathcal{T}$ and σ produce a change in the system, but each is required in order to define the changed system.*

Now that each parameter has been defined and described, let's take another look at the definition of General System:

$$\mathcal{G} =_{df} (\mathcal{A},\ \mathcal{P},\ \mathcal{T},\ \mathscr{T},\ \sigma)$$

Here we do need a refinement of the definition to provide greater relatedness of these parameters. That is, an additional parameter must be introduced. The definition is changed as follows:

$$\mathcal{G} =_{df} (\mathcal{A},\ \mathcal{P},\ \mathcal{Q},\ \mathcal{T},\ \mathscr{T},\ \sigma)$$

In this definition, another parameter has been added—$\mathcal{Q}$ (qualifiers, normally for component-qualifiers). With this modification, we now have the following definition:

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[45] Frick, T. (1990). "Analysis of Patterns in Time (APT): A Method of Recording and Quantifying Temporal Relations in Education." *American Educational Research Journal*, 27(1), 180-204.

General System Defined:

> ***General System*** $=_{df}$ a set of affect-relations ($\mathcal{A}$) which determine a set of partitioned components ($\mathcal{P}$) defined by component-qualifiers ($\mathcal{Q}$, a transition functions set ($\mathcal{T}$), a linearly-ordered time set ($\mathcal{T}$), and a state-transition function (σ).

This definition is formalized as follows:

$$\mathcal{G} =_{df} [\mathcal{A} \Vdash (\mathcal{P}\,(\mathcal{Q},\, \mathcal{T},\, \mathcal{T}, \sigma))]; \text{ where}$$

'⊩' is read "determines" or "which determine" or "from which is/are derived", as appropriate for the sentence in which it is used. This symbol is similar in intent to the logical "yields", but whereas "yields" is a logical relation for a deductive proof, this is a predicate relation identifying that which is derived from the existent set.

That is, a system is first recognized by the affect-relations and **not** the components of the system, as is commonly assumed. From the affect-relations, the partitioned components are obtained. Then the component-qualifiers are determined; that is, the properties of the system. Then the relatedness of the components through the transition functions is determined, a time assigned, and the state-transition is determined.

Now, let's take a look at the elements of each of these sets.

The sets that define $\mathcal{G}$ have the following elements:

$\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \in \mathcal{A}$;

$T_p, I_p, F_p, O_p, S_p, L_p, \mathcal{S}_{BX}, \mathcal{S}'_{BY} \in \mathcal{P}$; that is,
toput, input, fromput, output, storeput, spillput, system logistic qualifier, negasystem logistic qualifier, system background components, and negasystem background components.

$\mathcal{L}, \mathcal{L}' \in \mathcal{Q}$; that is, $\mathcal{L}$ are the system qualifiers, and $\mathcal{L}'$ are the negasystem qualifiers.

$f_{\mathcal{I}}, f_{\mathcal{O}}, f_{\mathcal{T}}, f_{\mathcal{B}}, f_{\mathcal{E}}, f_{\mathcal{S}} \in \mathcal{T}$; that is, feedin, feedout, feedthrough, feedback, feedintra, and feedstore.

$t_1, t_2, \ldots, t_k \in \mathcal{T} \subset {}_{T_p}\mathcal{L}$

Let the object-set of a *General System*, $\mathcal{G}_O$, be such that $\mathcal{G}_O = \mathcal{S}_X \cup \mathcal{S}'_Y$; where $\mathcal{S}_X$ and $\mathcal{S}'_Y$ are the object-sets of $\mathcal{S}$ and $\mathcal{S}'$, respectively. Then, $\mathcal{G}_O$ is defined by the following:

$$\mathcal{G}_O =_{df} \mathcal{S}_X \cup \mathcal{S}'_Y = (I_p \cup F_p \cup S_p \cup \mathcal{L} \cup \mathcal{S}_{BX}) \cup (T_p \cup O_p \cup L_p \cup \mathcal{L}' \cup \mathcal{S}'_{BY}).$$

Further, as all of these sets are disjoint, the following holds:

$$I_p \cap F_p \cap S_p \cap \mathcal{L} \cap \mathcal{S}_{BX} \cap T_p \cap O_p \cap L_p \cap \mathcal{L}' \cap \mathcal{S}'_{BY} = \emptyset.$$

Diagram of System Properties: The following graph characterizes the properties of a General System.

Diagram of System Properties

Universe of Discourse ($\mathcal{U}$) $\mathcal{G} =_{df} (\mathcal{A}, \mathcal{P}, \mathcal{Q}, \mathcal{T}, \mathcal{T}, \sigma)$ [1,2]

Basic Properties

System Environment or Negasystem ($\mathcal{S}'$)

(Logistic Control Predicates)

$\mathcal{S}$-Qualifiers ($\mathcal{L}$)

System ($\mathcal{S}$)

(Logistic Control Predicates)

$\mathcal{S}'$-Qualifiers ($\mathcal{L}'$)

Structural Properties

Storeput (S_p)

Toput (T_p) Input (I_p)

Fromput (F_p) Output (O_p)

Feedstore ($f_{\mathcal{S}}$)

Feedin (f_I)

Feedout (f_O)

Filtration $\mathcal{F}(\mathcal{S}) \subset {}_{T_p}\mathcal{L}$

Regulation $\mathcal{R}(\mathcal{S}) \subset {}_{F_p}\mathcal{L}'$

Feedintra ($f_{\mathcal{S}}$)

Feedthrough ($f_{\mathcal{S}}$)

Feedback ($f_{\mathcal{B}}$)

Feedenviron ($f_{\mathcal{E}}$)

Spillage

Dynamic Properties

[1] $\mathcal{G}$ is the *General System*, $\mathcal{A}$ is the *Family of Affect Relations Set*, $\mathcal{P}$ is the *Object Partitioning Set*, $\mathcal{Q}$ is the *Qualifier Set*, $\mathcal{T}$ is a *Transition Functions Set*, $\mathcal{T}$ is a *Linearly-Ordered Time Set*, and σ is the *System State Transition Function*.

[2] $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \in \mathcal{A}$; $T_p, I_p, F_p, O_p, S_p, L_p, \mathcal{S}_{BX}, \mathcal{S}'_{BY} \in \mathcal{P}$; $\mathcal{L}, \mathcal{L}' \in \mathcal{Q}$;

$f_{\mathcal{S}}, f_O, f_{\mathcal{S}}, f_{\mathcal{B}}, f_{\mathcal{E}}, f_{\mathcal{S}} \in \mathcal{T}$; and $t_1, t_2, \ldots, t_k \in \mathcal{T}$.

Summary

In this report we have discussed theories and models, and how *ATIS* is developed as an *axiomatic formal empirical theory* designed specifically for *intentional systems*.

Theories of learning were discussed and why it is that they do not and cannot provide a comprehensive and consistent theory, and why hypothesis-based development cannot result in a theory of learning.

We discussed the distinction between scenario-based and logic-based models and why a logic-based model is required if actual predictive outcomes are desired when studying educational systems or other types of intentional systems.

We also discussed how an empirical theory must be developed and why a logico-mathematical theory is required in order to gain a real understanding in the social sciences.

A definition of general system was presented and was given an initial formal development.